In this dissertation, we discuss designs of social security system, including public pension, child benefit programs, and some family policies, in models with individual fertility decisions. We believe that the discussion is important because the social security system and the fertility decisions are closely related.

In most developed countries, fertility rates have been declining and governments have made reforms in the social security system to cope with such a demographic change. In order for the reforms to be successful, we must have good understanding on the relationship between the social security system and the fertility.

Although the fertility rate was usually assumed to be exogenous in the classical literature on the social security system, there is a growing literature on the optimal social security system in the models with endogenous fertility. This dissertation intends to contribute to the rapidly growing field of studies.

The structure of this dissertation is as follows. After a comprehensive review of the literature related to the dissertation, we outline the original contributions of the dissertation in Chapter 1. The optimal financing method of public pension is studied in Chapter 2. The optimal social security level in an endogenous fertility model is examined Chapter 3. The condition for efficiency improvement by child benefit financed by child benefit is considered in Chapter 4. The optimal taxation rule in a household production model is derived in Chapter 5.

**Optimal Social Security Financing Method**

In the discussion of the social security system, the financing method has been one of the most important agenda. There are several possible methods to finance the social security system such as the Value-Added-Tax (VAT from now on) and the proportional payroll tax. Since the financing method affects the allocation in the economy, it is necessary to investigate which method can attain a higher welfare level.

In this study, we evaluate at which level the government should implement an intergenerational transfer through social security system and which allocation achieves a
higher welfare level using the parameters calibrated for the Japanese economy. We an-
swer to the above questions in an 80-period overlapping generation model with no private
annuity market, borrowing constraints, and uncertainty in employment. In the model,
an unemployment insurance is provided by government.

As a result, it is shown that the optimal pension level for each case is zero. This result
is consistent with the existing studies, such as Imrohoroğlu et al. (1995, 1999) and Fuster
et al. (2003). The result implies that increasing social security has a negative impact on
welfare.

Further, we consider a case where social security system is necessary for some reasons
such as myopic individuals. In such a case, welfares for both cases are not comparable
since the welfare under VAT method does not always dominate the one under payroll
tax method. In order to cope with this problem, we introduce land into the production
sector, then it is verified that the welfare is better under the VAT method than under
the payroll tax method given the parameters we assume.

The result comes from the following fact: the VAT method does not prevent capital
accumulation so much as the payroll tax. This factor makes the VAT method better.
While, the payroll tax serves as an unemployment insurance. This is a virtue of the
payroll tax method in this model. Moreover, the two methods have different effects on
the borrowing constraints. All those effects taken into consideration, quantitatively, the
result indicates that the VAT method is better than the payroll tax method in this study.

Optimal Social Security under Endogenous Fertility

The population growth rate determines the rate of return from social security system. The
population growth rate is, however, determined by households’ fertility decision making.
From the viewpoint of social security, if there are public pension systems, individuals
have no motive for having a child as income source for their life after retirement, or
they may change the number of children they have if they face borrowing constraints.
Therefore the social security system may affect households’ fertility decisions. Hence,
it is important to take into account an interaction between social security system and
population growth rate when the optimal social security level is considered.

This study considers the optimal social security level in a model with endogenous
population growth. In this model, households choose the number of children for obtaining
utility for themselves, where children are considered as consumption goods. This study
contributes to the literature on the optimal social security level. Although the issue of
dynamic efficiency is closely related to the population growth rate, the literature has not taken account of this factor well.

It is shown that the optimal payroll tax rate is 0% in the benchmark case, which result is consistent to the literature. Further, the optimal payroll tax rate is lower than that of the exogenous case in a setting where the optimal social security level is greater than 0% because of dynamic inefficiency.

The result shows that the population growth rate of the endogenous case is lower than that of the exogenous case and the optimal social security level is lower in the endogenous case. These results are interpreted as follows. There are properties of children such as requiring parents to make a certain amount of payment for many consecutive periods and giving no chance to be free from rearing children. The above properties let households avoid to have children when they do not have enough liquidity, which lowers the fertility rate as social security level rises, and the optimal social level is lowered in the endogenous case. There is also a factor that fertility rate is increased by an increase in social security level when the economy is dynamically inefficient. This result implies that the effect of lowering fertility rate dominates that of raising fertility rate. Since the rate of return from social security system financed by pay-as-you-go system hinges on the population growth rate directly, this factor causes the optimal social security level to be lower.

**Optimal Child Benefit Scheme in an Overlapping Generations Model**

This chapter studies a possibility of efficiency improvement by child benefit programs financed by newly issued debt in an endogenous fertility setting. We consider an overlapping generations economy with a certain amount of government debt accumulation. The literature shows that introducing child benefit system can improve efficiency in the economy in which there exists an externality for inter-generational transfer such as social security system. Further, Michel and Wigniolle (2007) showed that the possibility of efficiency improvement in the economy in which the capital is under-accumulated in an endogenous fertility model, but an explicit condition for the improvement is not considered there.

First of all, we consider a simple two periods model to get an intuition for efficiency improvement by introducing a child benefit program financed by newly issued bond. In the model, production is not taken into account and the factor prices are constant. If there is no government debt in this period, any increase of debt for child benefit will make future generations’ welfare worse since their per capita burden will increase no matter
how many children parents have. With a huge amount of debt, child benefit financed by additionally issued debt may improve the welfare of future generations if households increase the number of children enough. However, in a setting where factor prices change to an increase in child benefit, it requires more conditions for the improvement.

We derive conditions for efficiency improvement by introducing child benefit program using an efficiency criterion for an endogenous fertility setting. It is shown that the result crucially depends on the relative amount of accumulated government debt in the economy. Moreover, the condition is different depending on the change in per capita capital to an increase in child benefit. When per-capita capital increases to an increase in child benefit, the interest rate should drop, and thus the term of substitution effect needs to be satisfied. The reason is that the decline of capital income may be big enough so that the welfare may become worsened. On the other hand, the term of income effect must be satisfied when per-capita capital decreases to an increase of child benefit. The reason is that the decline in wage income may exceed the effect of tax burden decrease. This study contributes to a literature of optimal child benefit program in the setting where the government debt is accumulated.

**Optimal Taxation under Endogenous Fertility Decisions**

An optimal child benefit program has been considered in the literature in various settings with social security. It has not yet been well considered, however, in an optimal indirect taxation framework with leisure. Children have special properties that they are unavailable in the market and that they need time to be taken care of. Focusing on these properties, we explore the optimal indirect taxation rules in a model with leisure and household production for children.

We use both the primal approach and the dual approach to derive the optimal taxation rule. The primal approach allows us to compare the market equilibrium conditions to the optimal conditions without losing important information. The dual approach, however, gives us the optimal taxation rule in the elasticity form, which may provide a different implication from the optimal taxation rule.

In the model, we assume two types of untaxable time, leisure and child-rearing time. The child-rearing time is used as an input of household production function, which produces the amount of children. The other input is nursery service and that is taxable.

The existing literature has not focused on the possibility of taxation on the final goods since they are not observable in most cases. We consider the possibility since the
amount of children is observable if it represents the number. We check both observable and unobservable cases in the study.

In the case without rent, the derived optimal taxation rules suggest that nursery service should not be taxed if children are taxable. However, it should be taxed if the amount of children is untaxable. In the case with rent, however, nursery service should be taxed for maximizing the rent from household production activity. In this case, the tax rate of nursery service can be negative.

Further, we consider the case of Leontief household production. In such a case, if the household production is homogeneous of degree one, it is possible to tax goods as if lump-sum tax is available. This case corresponds to the setting of Kleven (2004), which derived the optimal taxation rule known as the inverse factor share rule. When the household production technology is not homogeneous of degree one, such a property does not hold any more.