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Abstract

Most of the increased productivity and social wealth in the world over the recent decades can be traced largely to technological innovation. Governments encourage firms’ innovation by providing R&D subsidies and other measures. At the same time, governments regulate technological advancement from the social safety standpoint through various legal and/or institutional measures. Typically, product liability scheme is legislated to assure that these advances do not pose a risk of harm to public. However, there is a public debate that product liability imposes costs that hit new products particularly hard, thus reducing firm’s incentives to innovate and their willingness to bear risk.

This research attempts to describe the government policies – innovation linkage as a quantitative model. In particular, this research employs Evolutionary Game Theory to model and simulate the dynamic behavior of the system where governments interact over the time with firms, then analyzes the impacts of typical government policies, i.e. product liability law and R&D subsidies. The results show some useful implications to achieve balanced policy goals between public safety and technological advancement.

Keywords: Evolutionary game theory, innovation, product liability

1. Introduction

Most of the increased productivity and social wealth in the world over the recent decades can be traced largely to technological innovation. Technological innovations are the creation of better or more effective products, processes and significant technological changes of products and processes that are accepted by markets, government and society (Wikipedia). Innovation is a key determinant of economic growth and an important source of social benefits. From the society standpoint, innovation aids in comfort, convenience, and improved safety-enhancing life. Today’s technological innovation for automobiles and airplane, for example, added to greater safety, maintenance, speed, and weight capacity for passenger services. Many vaccines have been developed and are used to prevent many various diseases and beneficial for public health.

From the firm’s standpoint, innovation has been recognized as a catalyst of the positive changes in efficiency, productivity, quality and competitiveness. Innovators could take market share
from non-innovators, growing faster, and ultimately being more profitable than non-innovators. On the other hand, innovativeness is associated with uncertainty and the risk of failure and tort liability (Leonard-Barton, 1988). Thus, while innovation increases the risk in the short run, it enhances the growth of firms in the long run (Herbert Dawid, 2001). These effects lead to the “innovator’s dilemma” (Christensen (1997)), the question, under which circumstances firms they should explore new technological innovation. The large body of research in this area can mainly be categorized into two groups. The first one adheres to an internal view and examines the impact of a firm’s internal capabilities on its strategies. The second group focuses on external forces that influence a firm’s strategy. In this line of research the impact of government policies on firm’s behavior is studied (Herbert Dawid, 2001).

Indeed, government policies play an important role in affecting the firm’s incentive towards innovation. Recognizing its noticeable effects on economic growth, quality of life, and productive growth, policymakers are working to develop policy environments that will foster innovation and its resulting positive social benefits. However, technological innovations are frequently accompanied by undesirable social or economic consequences, such as health or safety hazards. In these cases, the government as overseer and protector of the public interest must play a very direct role in controlling such undesirable effects, via planning, controls, regulations. Therefore, along with the policy instruments to encourage firm’s innovation, at the same time, governments regulate technological advancement from the social safety standpoint through various legal and/or institutional measures. Typically, product liability scheme is legislated to assure that these advances do not pose a risk of harm to public.

Product liability is the area of law in which manufacturers, distributors, suppliers, retailers, and others who make products available to the public are held responsible for the injuries those products cause. Although the word “product” has broad connotations, product liability as an area of law is traditionally limited to products in the form of tangible personal property (Wikipedia). Basically, strict liability and negligence are the two major competing rules of product liability used in current tort law. The main purpose of product liability is to provide firms with incentives to make product safer and to compensate those who are injured by failed products.

Product liability and innovation, by themselves, these term signal different disciplines. However, one of the most salient aspects of the impact of product liability has been its effect on innovation. Product liability ideally should promote efficient levels of product safety, but misdirected liability efforts may depress beneficial innovations (Viscusi, K.W., Moore, M.J, 1993). Indeed, there is recently a public debate that product liability imposes costs that hit new products particularly hard, thus reducing firm’s incentives to innovate and their willingness to bear risk. The
logic here has been straightforward: When the legal costs of certain kinds of accidents are prohibitively high and unpredictable, entire sectors of enterprise shut down. The statistic in U.S. case shows that from 1980s companies were experiencing more product liability cases and the size of the awards was also increasing. Consequently, corporate resources expended more on matters related to product liability, e.g. insurance costs had risen dramatically. Defective products may generate enormous legal liabilities, potentially undermining not only the profitability of the product but possibly the firm itself. Therefore, when an innovator cannot reduce this liability by improving the quality of her innovation, the effect of the law of torts on the incentive to innovate is perverse (Gideon Parchomovsky, Alex Stein, 2009).

The large uncertainty that remains about the effects of government policies concerning on the incentive of firm towards innovation. This may suggest to some further analysis on different aspects of government policies, both the fostering innovation aspect (e.g. by financial subsidies) and controlling innovation aspect (e.g. by product liability regulations). A list of questions should be answered to solve the current uncertainty of government policies in affecting new product development decisions: Do government subsidies have positive impacts on innovation activities of the firms? Whether product liability cost discourages innovation? How these policy instruments towards firm’s innovation can be combined by government? One clear thing is that effective policies call for a balancing of the incentives for improved product safety using product liability laws, and the benefits from innovation for both firms and society on the other.

In this research, we assume a simple model as shown in Figure.1. The model shows the strategic interaction between the government and the firms in the context of technological innovation. The government intervenes into the innovation process in two ways, by financial support for innovation (i.e. R&D subsidies) and by product liability regulation from product safety considerations.

![Figure.1: Innovation model](attachment:innovation_model.png)

In these circumstances, “The innovator’s dilemma” – “To Innovate or Not to Innovate” is an important decision of adaptive firms. If a firm decision in developing technological innovation leads to success, of course, firm can gain competitive advantage on that market. However, this decision
may also lead to the risk of tort liability when accidents occur for the users. Like any business plan, a firm decision of R&D spending on innovation is acted under private benefit-maximizing constraint, while government policy planner seeks to maximize social benefit and minimize the social cost due to the undesirable consequences from that innovation. Under the contrasting differences these two decision criteria, the interaction between governments and firms is happened as two competing interest groups of the game. Therefore, we model and simulate the interaction between governments and firms as two players of a game. In this game, firm chooses among alternative actions – to innovate or not to innovate - whose payoff depends on the different choices of policies by government. More specifically, we focus on the two competing aspects of government policies affecting on new product introductions by firms, namely, financial support (support subsidies or not to support subsidies), and product liability (strict liability or negligence). By analyzing these strategic game interactions, our study attempts to identify effectiveness and impact of government policies on the innovation incentive of manufacturing firms.

2. Research Methodology and Objectives

We interpret the game interactions between the government and the firm in the context of technological innovation in a dynamic approach by using evolutionary game theory (EGT).

EGT has grown into a field that combines the principles of game theory, evolution, and dynamical systems. Games agents are simulated with population interacting over time, with their behavior adjusting over time in response to the payoffs (utilities, profits). The focus of EGT study is the dynamic behavior of the system similar to natural selection of playing members of the populations to determine how the population evolves. The natural selection process that determines how populations playing specific strategies evolve is known as the replicator dynamics. The basic idea is that more successful behavior (i.e. more “fit”) tends to be more prevalent and less fit behaviors (Daniel Friedman, 1991). Ideally, under the dynamical process, the evolution would converge to some stable value for each population, which would represent a best response for each agent. Evolutionary game theory can be seen, in part, as filling an important lacuna of traditional game theory with the treatment of the dynamics of rational deliberation, to capture some of the dynamics of the decision-making process in traditional game theory.

This research attempts to describe the government policies – innovation linkage as a quantitative model. In particular, this research employs Evolutionary Game Theory theoretical background to model and simulate the dynamic behavior of the system where governments interact over the time with firms, then analyzes the impacts of typical government policies, i.e. product liability law and R&D subsidies. We assume that each individual (i.e. firm) strategy is
converging through the dynamic process under the social mechanism of learning and imitation. Based on the results of the evolution of the game, we analyze the role of different factors of government’s policies in affecting the new product development decisions of firms. The objective of this study is to find the optimal strategy for the implementation of government policies and its effects on the incentive for product innovation of firm. The results show some useful implications to achieve balanced policy goals between public safety and technological advancement.

3. Game Model Establishment

Consider the case in which a risk-neutral firm has to choose ex-ante whether or not to produce a new innovative product for the market, which can cause accidents to consumers. Firm’s expected benefit from innovation is affected by government policy environment. We suppose that both government and firm are reasonable self-interested decision makers, whose goal is to minimize their expected loss and maximize their expected utility. More specifically, government, by his policy instruments, tries to encourage the firm’s innovation for the benefit to society, as well as, to minimize the social cost due to the consequences posed by a new product to the public. For the firms, they try to maximize their expected revenue from the innovation and to minimize the expected liability cost as well.

Since the economic reality is too complicated to be analyzed in the evolutionary game model without simplifications, a number of assumptions is deemed necessary.

For the manufacturing firms, they might adopt strategy of innovating (strategy $F_1$) or not innovating (strategy $F_2$). New innovation is expected to not only give private benefit $B_P$ to the firm, but also generate the social benefit $B_S$. The innovation development requires high R&D cost $C$. It also has the risk of tort liability when the accident happens to the consumer. The risk of an accident to consumer is affected by firm’s choice of level of safety for his product. The firm can reduce the dangers associated with his innovative product by taking safety precaution. Safety precaution reduces expected accident cost but is costly to the firm. If we define $s$ as the variable of the safety level he spend for his product, $C(s)$ will be the function of $s$ expressing the relation between the cost of innovation and the safety level. The cost of producing innovative products is assumed to increase to the higher level of safety at an increasing rate, so that $C'(s) > 0$ and $C''(s) > 0$.

In this research, we only investigate the case that there is unilateral precaution in accident prevention. It is assumed that the consumer behavior cannot influence the amount of expected damages. Thus, the riskiness is a function only of the firm’s chosen level of safety $s$. Thus, riskiness of production is represented by the probability of having an accident $p(s) \in [0,1]$, which the
probability of an accident decreasing at a decreasing rate in the level of safety, \( p'(s) < 0 \) and \( p''(s) > 0 \). If accidents happen, the total damage loss is \( L \) (where \( L > 0 \)).

In this research, we propose two different cases of government policy environment towards firm’s innovation. In the first case, we consider solely the product liability aspect on the government-firm game results. Liability rules have been approved by many researches that they have important impacts on firm incentives on innovation and economic efficiency. There have been substantial interests in the product liability issue focusing on the effects of product liability on a firm’s incentives for ex ante innovation decision. Therefore, in this game model, we treat the government as a player who has two different strategies, namely, strict liability (strategy \( G_1 \)) or negligence (strategy \( G_2 \)). By solving the model where government policy focuses only on the safety standpoint, we analyze the impact of liability rules can on a firm’s innovation behavior. How firm incentive is generated when strict liability or negligence is the underlying liability scheme. The comparison between strict liability and negligence shows that punishment (i.e. product liability regulation) alone is insufficient for motivating firm doing innovation in positive ways. This result motivates us to propose another scheme for the government in the second case.

Thus, in the second case 2, we propose the combination of product liability instrument and financial subsidy instrument for the implementation of government policies toward firm’s innovation. In this game model, the government can choose to implement either the strict liability with the subsidies \( F \) (strategy \( G'_1 \)) or negligence without any subsidies (strategy \( G'_2 \)). The research then, justifies the case when both reward (i.e. financial support) and punishment (i.e. product liability) considered in the government policies.

The list of parameters of the game model is summarized in the following table:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s )</td>
<td>Safety precaution level for innovation</td>
</tr>
<tr>
<td>( C(s) )</td>
<td>Cost of innovation</td>
</tr>
<tr>
<td>( p(s) )</td>
<td>Product liability risk</td>
</tr>
<tr>
<td>( L )</td>
<td>Total damages caused by new innovative products</td>
</tr>
<tr>
<td>( B_P )</td>
<td>Private benefit for firm from innovative products</td>
</tr>
<tr>
<td>( B_S )</td>
<td>Social benefit from innovative products</td>
</tr>
<tr>
<td>( F )</td>
<td>Government financial support</td>
</tr>
</tbody>
</table>
To study the dynamic that arises from the interaction between government and the firm in evolution process, firms and governments are represented by a two-population in an evolutionary game model, where the population of firms strategically interacts with that of governments. Let us assume that at each period of time $t$ potential firm and government play a one-shot population game (i.e. All agents play the game simultaneously). We assume the two populations to be constant over the time and normalize to 1 the number of both potential firms and government. Let the variable $x(t)$ denote the share of government choosing the $G_1$ strategy (for the first model) or $G'_1$ strategy (for the second model) at time $t$, $0 \leq x(t) \leq 1$. Analogously, let $y(t)$ denote the share of potential firms adopting choice $F_1$ at time $t$, $0 \leq y(t) \leq 1$.

4. Model 1 – Product liability being the only instrument for the implementation of government policies toward firm’s innovation

4.1. Payoff Matrix of the Game Model 1

In this case, governments only consider the use of product liability as means of controlling accident risks. Firms may reduce the risk of tort liability for by taking care about the safety level for their innovative products; and because the risk varies depending on the level of safety they choose, their private benefit varies as well.

We establish the matrix payoff game as played by a row player – the government and a column player – the firm. The government as a row player chooses among two strategies: strict liability (strategy $G_1$) or negligence (strategy $G_2$); the firm as the column player simultaneously chooses whether to innovate (strategy $F_1$) or not to innovate (strategy $F_2$). The outcome of the game is a pair of payoffs where the first entry is the payoff of the row player (i.e. the government), and the second is the payoff of the column player (i.e. the firm).

In case that the strict liability is the underlying liability scheme chosen by the government, firm must pay for all accident losses his product causing, regardless of the extent of his safety care, which equals to $p(s)L$. In case of negligence, firm is held liable for accident his product causing only if he is judged by the court that he was negligent, that is, only if his safety care was less than the due safety standard $s_G$. If we call the probability of firm failure in suit as $\gamma(s)$, firm’s expected liability cost is $\gamma p(s)L$.

Without loss of generality, we can normalize to zero the payoff of firm deciding not to innovate.
Figure 2: Payoff Matrix of Model 1

4.2. Analysis of the Game Model 1

We denote by $x(t)$, $y(t)$ the frequency of strategy $G_i$ and $F_i$ respectively at the time $t$ and assume that these frequencies change according to the success of the strategies. The expected payoff of the government and the firm are given by:

Expected payoff of the government choosing “Strict Liability” strategy is:
$$E_{G_1} = yB_S + (1 - y)0 = yB_S$$

Expected payoff of the government choosing “Negligence” strategy is:
$$E_{G_2} = y[B_S - (1 - y)p(s)L] + (1 - y)0 = y[B_S - (1 - y)p(s)L]$$

The expected payoff of the firm choosing “Do Innovation” strategy is
$$E_{F_1} = x[B_S - C(s) - p(s)L] + (1 - x)[B_P - C(s) - \gamma p(s)L]$$

The expected payoff of the firm choosing “Not Do Innovation” strategy is:
$$E_{F_2} = x0 + (1 - x)0 = 0$$

The average payoffs of government and the firm are $\bar{E_G}$ and $\bar{E_F}$ respectively.

The average payoff of the government is: $\bar{E_G} = xE_{G_1} + (1 - x)E_{G_2}$

The average payoff of the firm is: $\bar{E_F} = yE_{F_1} + (1 - y)E_{F_2}$

We shall assume a particularly simple learning mechanism and postulate that the rate according to which a $i$ strategy chosen-player switches to a $j$ strategy is proportional to the payoff difference $E_{G_j} - E_{G_i}$ ($E_{F_j} - E_{F_i}$). The process of adopting strategies is modeled by the so called replicator dynamics (Weibull 1995), according to which the strategies with expected payoffs greater than the average payoff spread within the populations at the expense of the alternative strategies.

Replicator dynamics equation of government “Strict liability” strategy is:
Replicator dynamics equation of firm “Do Innovation” strategy is:
\[
\frac{dx}{dt} = x(E_{c_1} - \overline{E_c}) = x(1 - x)(E_{c_1} - E_{c_2}) = f(x, y)
\]
\[
\frac{dy}{dt} = y(E_{f_1} - \overline{E_f}) = y(1 - y)(E_{f_1} - E_{f_2}) = g(x, y)
\]
where
\[
f(x, y) = x(1 - x)y(1 - y)p(s)L
\]
\[
g(x, y) = y(1 - y)[-x(1 - y)p(s)L + (B_p - C(s) - \gamma p(s)L]
\]
Let us consider the dynamic system \[
\begin{align*}
\frac{dx}{dt} &= f(x, y) \\
\frac{dy}{dt} &= g(x, y)
\end{align*}
\]
which is defined in \([0,1]^2\), that is, in the unit square S:
\[
S = \{(x, y): 0 \leq x \leq 1, \ 0 \leq y \leq 1\}
\]

**Proposition 1:** The two dimensional system \[
\begin{align*}
\frac{dx}{dt} &= f(x, y) \\
\frac{dy}{dt} &= g(x, y)
\end{align*}
\]
has four equilibria on the boundary of \([0,1]^2\), i.e. the four vertices a. The three vertices (0,0), (1,0), (0,1) are unstable, while the vertex (1,1) is stable if \(B_p > C + pL\) (1)

**Proof:** The stability of four equilibria (i.e. fixed points) of the two dimensional system \[
\begin{align*}
\frac{dx}{dt} &= f(x, y) \\
\frac{dy}{dt} &= g(x, y)
\end{align*}
\]
can be obtained through the analysis of Jacobean matrix of the system as:
\[
J = \left[ \begin{array}{cc} \frac{\partial f(x, y)}{\partial x} & \frac{\partial f(x, y)}{\partial y} \\ \frac{\partial g(x, y)}{\partial x} & \frac{\partial g(x, y)}{\partial y} \end{array} \right] = \left[ \begin{array}{cc} -(1 - 2x) y(1 - \gamma) pL & x(1 - x)(1 - \gamma) pL \\ -y(1 - y)(1 - \gamma) pL & -(1 - 2y)[-x(1 - \gamma) pL + (B_p - C - \gamma pL] \end{array} \right]
\]

Evaluating the Jacobian at the equilibrium point (0,0), we get:
\[
J(0,0) = \begin{pmatrix} 0 & 0 \\ 0 & B_p - C - \gamma pL \end{pmatrix}
\]
The Jacobian at the equilibrium point (0,0) has 2 eigenvalues \(\lambda_1 = 0\) and \(\lambda_2 = B_p - C - \gamma pL\). Therefore, the equilibrium point (0,0) is unstable.

Evaluating the Jacobian at the equilibrium point (0,1), we get:
\[
J(0,1) = \begin{pmatrix} (1 - \gamma) pL & 0 \\ 0 & -B_p + C + \gamma pL \end{pmatrix}
\]
The Jacobian at the equilibrium point (0,1) has 2 eigenvalues \( \lambda_1 = (1 - \gamma) pL \) and \( \lambda_2 = -B_p + C + pL \). Among these two eigenvalues, \( \lambda_1 \) is real positive. Therefore, the equilibrium point (0,1) is unstable.

Evaluating the Jacobian at the equilibrium point (0,1), we get:

\[
J(1,0) = \begin{pmatrix}
0 & 0 \\
0 & B_p - C - pL
\end{pmatrix}
\]

The Jacobian at the equilibrium point (1,0) has 2 eigenvalues \( \lambda_1 = 0 \) and \( \lambda_2 = B_p - C - pL \). Therefore, the equilibrium point (1,0) is unstable.

Evaluating the Jacobian at the equilibrium point (1,1), we get:

\[
J(1,1) = \begin{pmatrix}
-(1 - \gamma) pL & 0 \\
0 & -B_p + C + pL
\end{pmatrix}
\]

The Jacobian at the equilibrium point (1,1) has 2 eigenvalues \( \lambda_1 = -(1 - \gamma) pL \) and \( \lambda_2 = -B_p + C + pL \). Therefore, the equilibrium point (1,1) is stable if:

\[-B_p + C + pL < 0 \text{ or } B_p > C + pL\]

### 4.3. Discussion

The game results show that the system has four equilibria. From the dynamics that emerge in the model, it turns out that only one of these four equilibria (i.e. the vertex (1,1)) can be the evolutionary stable strategy (ESS) if the condition (1) is satisfied. In this ESS, the government and the firm would be better-off at (1,1) in which all firms will decide to innovate under the strict liability regulation chosen by the government if they find that they still have positive benefit from their innovation activities (i.e. \( B_p > C(s) + \gamma p(s)L \)).

Comparing the condition (1) with the social welfare optimality condition that the government concerns about any innovation activity (that is the social benefit must be greater than the social cost, i.e. \( B_S + B_p > C(s) - p(s)L \)), it is clear that, social welfare optimality condition doesn’t coincide with private benefit condition. In other words, the strict liability rule does not necessarily lead to the socially efficient outcome. The intuition is quite clear. Firm just only care only her own benefit (i.e.\( B_p \)) regardless of the benefit for society (i.e.\( B_S \)). Consequently, a more risky activity (i.e. higher \( p(s) \)) is refused by firm. Social benefit that many innovations can bring is ignored under strict liability.
5. Model 2 - Combination of product liability instrument and financial subsidy instrument for the implementation of government policies toward firm’s innovation

5.1. Payoff Matrix of the Game Model 2

The first model shows that the strict liability rule may lead firm evolution not convergence in “To innovate” strategy if firm finds that the innovation is too risky to return private benefit. The government policies solely using strict liability and negligence show that punishment (i.e. product liability regulation) alone is insufficient for motivating firm doing innovation in positive ways. This result motivates us to propose another scheme for the government. In the model 2, we propose the combination of product liability instrument and financial subsidy instrument for the implementation of government policies toward firm’s innovation. We solve the game model when government policy towards firm’s innovation is trying to balance between public safety consideration and technological advancement motivation. The government tries to increase the innovation incentive for firm by giving the financial support $F$. The government as a row player chooses among two strategies: strict liability with subsidy (strategy $G’_1$) or negligence without subsidy (strategy $G’_2$); the firm as the column player simultaneously considers whether to innovate (strategy $F_1$) or not to innovate (strategy $F_2$).

![Figure 3: Payoff Matrix of Model 2](image)

5.2. Analysis of Game Model 2

We denote by $x(t)$, $y(t)$ the frequency of strategy $G’_1$ and $F_1$ respectively at the time $t$ and assume that these frequencies change according to the success of the strategies. The expected payoff of the government and the firm are given by:

The expected payoff of the government choosing “Strict Liability with Subsidy” strategy is:

$$E_{G’_1} = y(B_s - F) + (1 - y)0 = y(B_s - F)$$

The expected payoff of the government choosing “Negligence without Subsidy” strategy is:
The expected payoff of the firm choosing “Do Innovation” strategy is
\[ E_{F_1} = x[B_S - C(s) - p(s)L + F] + (1-x)[B_p - C(s) - \gamma p(s)L] \]

The expected payoff of the firm choosing “Not Do Innovation” strategy is:
\[ E_{F_2} = x0 + (1-x)0 = 0 \]

The average payoffs of the government and the firm are \( \bar{E}_{G^t} \) and \( \bar{E}_F \) respectively.

The average payoff of the government is:
\[ \bar{E}_{G^t} = xE_{G_1} + (1-x)E_{G_2} \]

The average payoff of the firm is:
\[ \bar{E}_F = yE_{F_1} + (1-y)E_{F_2} \]

The evolution of firm’s and government’s behavior is modeled in the paper using the replicator dynamics.

Replicator dynamics equation of government “Strict Liability with Subsidy” strategy is:
\[
\frac{dx}{dt} = x(E_{G_1} - \bar{E}_{G^t}) = x(1-x)(E_{G_1} - E_{G_2}) = h(x,y)
\]

Replicator dynamics equation of firm “Do Innovation” strategy is:
\[
\frac{dy}{dt} = y(E_{F_1} - \bar{E}_F) = y(1-y)(E_{F_1} - E_{F_2}) = l(x,y)
\]

where
\[
h(x,y) = x(1-x)y[-F + (1-\gamma)p(s)L] \]
\[
l(x,y) = y(1-y)\{[F - (1-\gamma)p(s)L] + (B_p - C(s) - \gamma p(s)L)\}
\]

Let us consider the dynamic system 4 which is defined in \([0,1]^2\), that is, in the unit square \( S \):
\[ S = \{(x,y): 0 \leq x \leq 1, \ 0 \leq y \leq 1\} \]

**Preposition 2:** System \( \begin{cases} \frac{dx}{dt} = h(x,y) \\ \frac{dy}{dt} = l(x,y) \end{cases} \) has four equilibria on the boundary of \([0,1]^2\), i.e. the four vertices. The two vertices \((0,0)\) and \((1,0)\) are unstable. Moreover the vertex \((0,1)\) is stable if
\[
\frac{p(s)L-F}{p(s)L} < \gamma(s) < \frac{B_p-C(s)}{p(s)L} \] (2), while the vertex \((1,1)\) is stable if \( F > C(s) + p(s)L - B_p \) (3)

**Proof:** The stability of four equilibria (i.e. fixed points) of the two dimensional system
\[
\begin{cases} \frac{dx}{dt} = h(x,y) \\ \frac{dy}{dt} = l(x,y) \end{cases}
\]
can be obtained through the analysis of Jacobean matrix of the system as:
\[
J = \begin{pmatrix}
\frac{\partial h(x, y)}{\partial x} & \frac{\partial h(x, y)}{\partial y} \\
\frac{\partial l(x, y)}{\partial x} & \frac{\partial l(x, y)}{\partial y}
\end{pmatrix}
\]

Evaluating the Jacobian at the equilibrium point (0,0), we get:
\[
J(0,0) = \begin{pmatrix}
0 & 0 \\
0 & B_p - C - \gamma pL
\end{pmatrix}
\]

The Jacobian at the equilibrium point (0,0) has 2 eigenvalues \( \lambda_1 = 0 \) and \( \lambda_2 = B_p - C - \gamma pL \). Therefore, the equilibrium point (0,0) is unstable.

Evaluating the Jacobian at the equilibrium point (0,1), we get:
\[
J(0,1) = \begin{pmatrix}
(1 - \gamma) pL - F & 0 \\
0 & -B_p + C + \gamma pL
\end{pmatrix}
\]

The Jacobian at the equilibrium point (0,1) has 2 eigenvalues \( \lambda_1 = (1 - \gamma) pL - F \) and \( \lambda_2 = -B_p + C + \gamma pL \). Therefore, the equilibrium point (0,1) is stable if these two eigenvalues are both real negative, given by:
\[
\frac{pL - F}{pL} < \gamma < \frac{B_p - C}{pL}
\]

Evaluating the Jacobian at the equilibrium point (1,0), we get:
\[
J(1,0) = \begin{pmatrix}
0 & 0 \\
0 & B_p - C - pL + F
\end{pmatrix}
\]

The Jacobian at the equilibrium point (1,0) has 2 eigenvalues \( \lambda_1 = 0 \) and \( \lambda_2 = B_p - C - pL + F \). Therefore, the equilibrium point (1,0) is unstable.

Evaluating the Jacobian at the equilibrium point (1,1), we get:
\[
J(1,1) = \begin{pmatrix}
-(1 - \gamma) pL + F & 0 \\
0 & -B_p + C + pL + F
\end{pmatrix}
\]

The Jacobian at the equilibrium point (1,1) has 2 eigenvalues \( \lambda_1 = -(1 - \gamma) pL + F \) and \( \lambda_2 = -B_p + C + pL + F \). Therefore, the equilibrium point (1,1) is stable if these two eigenvalues are both real negative, i.e. \( C + pL - B_p < F < (1 - \gamma)pL \).

5.3. Discussion

The game results show that the system has four equilibria. From the dynamics that emerge in the model, it turns out that two of these four equilibria (i.e. (0,1) and (1,1)) can be the ESS if the condition (2) and (3) are satisfied respectively.
In the (0,1) ESS, under condition (2), the evolution is that all firms will decide to innovate under the negligence regulation even without any financial support by the government. By analyzing the condition (2), we can find that there exists the lower $s_1$ and upper $s_2$ boundaries for safety level that firm choose for their product. Therefore, if governments would like to regulate negligence as the product liability scheme, they should set the due safety standard that satisfies these boundaries (i.e. $s_1 < s_G < s_2$). Out of these boundaries, (0,1) will not an ESS anymore. The intuition is quite clear. If $s_G \geq s_2$, the too high safety standard required by government will discourage firm to do innovation. However, $s_G \leq s_1$ means the too low safety standard will not assure the social optimality.

![Figure 4: The condition (2)](image)

In the (1,1) ESS, under condition (3), the evolution is that all firms will decide to innovate under the strict liability regulation if the government's subsidy assure that firm will not face loss when doing innovation. The solution to give firms more incentive towards innovation by support the subsidy $F > C(s) + p(s)L - B_p$. This scheme may work especially for some kinds of innovation with high social payoffs but the industrial sector itself has the high risk to be loss. In these cases, the government has to consider giving them the subsidy with the relevant amount.

To sum up, in this second case, both strict liability and negligence can work effectively as they motivate firm’s innovation if the governments adjust some measures appropriately.

6. Conclusions

This research describes the government policies – innovation linkage by employing Evolutionary Game Theory to model and simulate the dynamic behavior of the system where governments interact over the time with firms, then analyzes the impacts of typical government policies, i.e. product liability law and R&D subsidies.

We propose two different cases of government policy environment towards firm’s innovation. In the first case, we consider solely the product liability aspect on the government-firm game results.
By solving the model where government policy focuses only on the safety standpoint, we analyze the impact of liability rules on a firm’s innovation behavior. The results show that firm incentive to innovate may be generated when strict liability is the underlying liability scheme. However, this scheme may not necessarily lead to the socially efficient outcome when the private benefit is not assured. Consequently, an innovation with the high social benefit but risky may be refused by the firm. Social benefit that many innovations can bring is ignored under strict liability in this case. It is clear that the punishment alone are insufficient for motivating firm doing innovation in positive ways. This result motivates us to propose another scheme for the government. In the second case 2, we propose the combination of product liability instrument and financial subsidy instrument for the implementation of government policies toward firm’s innovation. The research then, justifies the case when both reward (i.e. financial support) and punishment (i.e. product liability) considered in the government policies. The results show that both strict liability and negligence can work effectively towards firm’s innovation if the governments adjust the adding measures about the financial subsidies and safety standards appropriately. The results may lead to some useful implications to achieve balanced policy goals between public safety and technological advancement of firm’s innovation.

References


