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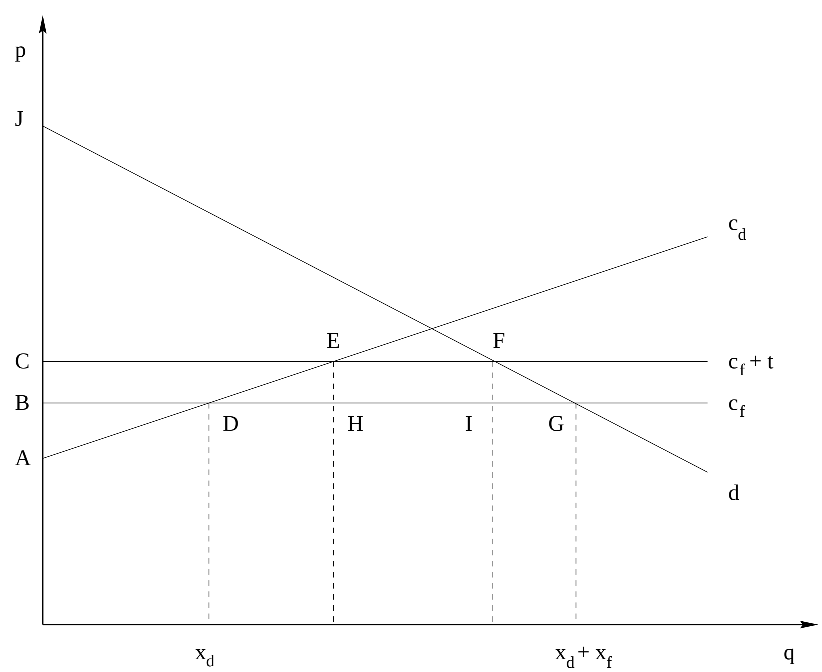
Economics 603
Micro III

Solution Quiz #1

(1) Suppose that a (large) number of domestic and foreign firms produce good x. All foreign firms have the same convex cost function $c_f(q)$ and all domestic firms have the same strictly convex cost function $c_d(q)$. The government is considering imposing a per-unit tariff of τ on imports of good x. Assume that partial equilibrium analysis is valid.

(a) Show that if c_f exhibits constant returns to scale, then imposing a small tariff lowers domestic welfare.

(b) Show that if c_f is strictly convex, then imposing a sufficiently small tariff raises domestic welfare.



(2) In a two-consumer, two-commodity pure exchange economy with continuous, strictly convex and strongly monotone preferences, consider the effects on

consumer 1's welfare of changes in the initial endowments $\omega_1 = (\omega_{11}, \omega_{21})$ and $\omega_2 = (\omega_{12}, \omega_{22})$.

(a) Suppose that the preferences of both consumers are quasilinear with respect to good 1. Consider a transfer from consumer 2 to consumer 1: $\hat{\omega}_1 = \omega_1 + z$ and $\hat{\omega}_2 = \omega_2 - z$ with $z \geq 0$. Show that the utility of consumer 1 cannot decrease.

(b) Assume now that the consumers' preferences are general (not necessarily quasilinear). Consider the same type of transfer in (a). Show that it is possible for the utility of consumer 1 to decrease.

(3) Consider a two-consumer, two-good, two-input (*closed*) economy. The productions function are

$$x = (k_x)^{\frac{1}{2}}(\ell_x)^{\frac{1}{2}} \quad \text{and} \quad y = 4(k_y)^{\frac{1}{2}}(\ell_y)^{\frac{1}{2}}.$$

The utility functions are

$$u_1(x_1, y_2) = x_1 y_1^2 \quad \text{and} \quad u_2(x_2, y_2) = x_2^2 y_2.$$

The initial endowments are

$$(K_1, L_1) = (800, 100) \quad \text{and} \quad (K_2, L_2) = (400, 200).$$

Find the competitive equilibrium for this economy.

Since the firms have CRS technology, the firms do not make profits in equilibrium and it doesn't matter who owns the firms. Firm 1, for example, solves the following optimization problem:

$$\max\ p_x \sqrt{k_x \ell_x} - r k_x - w \ell_x.$$

The FOC for this problem are

$$\frac{p_x}{2}\sqrt{\frac{\ell_x}{k_x}} = r \quad \text{and} \quad \frac{p_x}{2}\sqrt{\frac{k_x}{\ell_x}} = w.$$

Therefore,

$$\frac{\ell_x}{k_x} = \frac{r}{w} = \frac{\ell_y}{k_y} \quad \text{or} \quad w\ell_x = r k_x \quad \text{and} \quad w\ell_y = r k_y.$$

Since $\ell_x + \ell_y = L_1 + L_2 = 300$ and $k_x + k_y = K_1 + K_2 = 1200$, adding up the last two equations above we get that $300w = 1200r$. WLOG, make $r = 1$ (capital is the numeraire). Then $w = 4$ and $k_x = 4\ell_x$ and $k_y = 4\ell_y$. Therefore

$$p_x = 2r\sqrt{\frac{k_x}{\ell_x}} = 4 = 2r\sqrt{\frac{k_y}{\ell_y}} = p_y.$$

Let $b_1 = 800r + 100w = 1200$ and $b_2 = 400r + 200w = 1200$ be the budgets of the consumers. The consumers have Cobb-Douglas preferences. Therefore

$$x_1 = \frac{b_1}{3p_x} = 10, \quad y_1 = \frac{2b_1}{3p_y} = 20, \quad x_2 = \frac{2b_2}{3p_x} = 20, \quad y_2 = \frac{b_2}{3p_y} = 10.$$

Thus, the firms produce $x = x_1 + x_2 = 30$ and $y = y_1 + y_2 = 30$ respectively.