Econometrics I: Final Examination

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Answer all of the following questions. Use separate sheets of paper to answer questions. Clearly indicate which problem you answer. Sort your answer sheets in the order of question number and staple them when you submit. Write legibly. Illegible answer won' be graded.

1 Problem 1: Basic Questions

You attempt to estimate the following model:

$$y = x\beta + u, \tag{1}$$

where x is a $(1 \times k)$ vector of explanatory variables and β is a $(k \times 1)$ vector of parameters. We assume $\text{rank}\{Ex'x\} = k$. We also assume that i.i.d. sample from the population is available.

1-1. What is an additional condition for the OLS estimator $\hat{\beta}$ to be consistent? (5 pts)

1-2. Show the derivation of the OLS estimator $\hat{\beta}$ under the assumption given in 1-1. (5 pts)

1-3. Using the weak law of large number, show the consistency of the OLS estimator $\hat{\beta}$ under the assumption given in 1-1. (5 pts)

1-4. Using the central limit theorem, derive the asymptotic distribution of the OLS estimator $\hat{\beta}$. (5 pts)

1-5. Evaluate the following claim. "If the error term u is not normally distributed, the OLS estimator $\hat{\beta}$ is not normally distributed." (5 pts)

1-6. Use the iterated law of expectations to show the unbiasedness of the OLS estimator (i.e. $E(\hat{\beta}|x) = \beta$) under the assumption $E(u|x) = 0$. (5 pts)

1-7. What is the benefit of showing the unbiasedness of an estimator over showing its consistency. (5 pts)

1-8. Write down the definition of heteroscedasticity. What problem does the presence of it cause for the OLS estimator? (5 pts)

1-9. Explain what is multicolinearity. What problem does the presence of it cause for the OLS estimator? (5 pts)

1-10. Suppose that there is a $(1 \times k)$ vector of instrumental variables z that satisfies the assumptions $Ez'u = 0$ and $\text{rank}\{Ez'x\} = k$. We know that the IV estimator $\hat{\beta}^{IV}$ is also a consistent estimator. Explain why the OLS estimator is preferred to the IV estimator when the condition given in 1-1 holds. (5 pts)

2 Problem 2: Standard Questions

A development economist attempts to estimate the demand function of rice in Indian villages. The demand function of each household is given as

$$q_{ij}^{d} = \beta_0 + \beta_1 p_j + u_{ij}, \tag{2}$$

where q^d is rice demand, p is rice price, i is an index for each household and j is the index for each village. As implied from the subscript, all villagers in village j faces the same rice price. The error term u_{ij} is decomposed into two parts, $u_{ij} = c_j + e_{ij}$, where c_j is the taste shifter that is shared by all villagers in a village j and e_{ij} is the idiosyncratic shock to the taste that is uncorrelated with p_j. Both error term has zero expected values (i.e. $Ec_j = 0$ and $Ee_{ij} = 0$). We assume that there are no transaction of rice across villages due to bad road conditions. Thus the supply and demand of rice should equilibriate in each village. Denoting *per household* quantity of rice supply as q_j^s, which is purely determined by the weather of the village j but does not respond to price, the equilibrium condition is given as

$$q_j^s = q_j^d = \beta_0 + \beta_1 p_j + c_j \tag{3}$$

using the fact that the sample size for each village is large enough so that the law of large number applies and we can neglect e_{ij} because $\lim_{n\to\infty}(1/n)\sum_{i=1}^{n} e_{ij} = Ee_j = 0$. From this equilibrium condition, the price of rice in village j is given as

$$p_j = (q_j^s - \beta_0 - c_j)/\beta_1. \tag{4}$$

As mentioned before, we assume that q_j^s is purely determined by the weather in village j denoted as z_j and the weather is uncorrelated with u_{ij}. The development economist collected random household sample from several villages in India.

2-1. Suppose (2) is estimated via OLS, calculate the probability limit of the OLS estimator $\hat{\beta}$. (5pts) (HINT: Calculate the probability limit of the OLS estimator leaving p_j in the formula at first and substitute (4) into the "bias" term.)

2-2. Is the OLS estimator $\hat{\beta}$ consistent? If not, discuss the sign of the asymptotic bias. Answer this question based on the probability limit that you calculated in 2-1. (5pts)

2-3. Use the information given in the problem, suggest an IV estimation strategy to obtain a consistent estimator of β_1. Explain very clearly but concisely so that the development economist can follow your instruction. (5pts)

2-4. Concretely explain how the development economist should check that the instrumental variable is partially correlated with the endogenous variable. (5pts)

2-5. Suppose that there are two measures of weather conditions, the amount of rail fall in a village z_{1j} and the variability of village temperature z_{2j}. Clearly explain how the development economist can check whether at least an instrumental variables are truly exogenous given the other instrumental variable is exogenous. (5pts)

2-6. Suppose that high rainfall somehow reduces villagers' apatite, thus z_{1j} and u_{ij} are negatively correlated. If high rainfall increases a village rice supply quantity, what is the direction of asymptotic bias of the IV estimator when only z_{1j} is used as the instrumental variable. (5pts)

3 Problem 3: Advanced Questions

Neoclassical economic growth theory a la Kenneth Solow predicts that poorer countries grow faster than richer countries. To empirically test this hypothesis, you attempt to estimate the following equation.

$$\ln(y^*_{i2004}/y^*_{i1965}) = \beta_0 + \beta_1 \ln y^*_{i1965} + u_i, \tag{5}$$

where y^*_{it} is the true value of GNP of country i in year t. We assume that $Ey^*_{i1965}u_i = 0$. The neoclassical theory predicts that $\beta_1 < 0$. Due to the data quality in 1965, the GNPs in 1965 are measured with error:

$$\ln y_{i1965} = \ln y^*_{i1965} + e_i, \tag{6}$$

where $\ln y_{i1965}$ is the measured GNP and e_i is the classical measurement error (i.e. $\ln y^*_{i1965}$ and e_i are uncorrelated). On the other hand, GNPs in 2004 are measured without error (i.e. $\ln y_{i2004} = \ln y^*_{i2004}$).

3-1. Derive the probability limit of the OLS estimator $\hat{\beta}_1$ that is obtained by regressing $\ln(y_{i2004}/y_{i1965})$ on a constant and $\ln y_{i1965}$. Does this procedure render a consistent estimator of β_1? If not, discuss the sign of the asymptotic bias. (10pts)

3-2. Suggest a way to obtain a consistent estimator of β_1. Give concrete suggestions; for example, what kind of data you suggest to use, what estimator to use, and so on. (10pts)