

Midterm Examination

Suggested Answer

Name _____

Student # _____

Answer each question in the space provided. Your answer must be in **English**. You are allowed to use a dictionary and a calculator. Please write legibly so that I can read your answer. This examination ends at 9:55.

1. Questions on Household Utility Maximization (50pts intotal)

Suppose a household maximizes its utility defined as $U(C, L)$ where C is weekly consumption amount and L is weekly hours of leisure. The marginal utility of consumption and leisure are both positive. The household has non-labor income, denoted as V , and disposable hours, denoted as T , which can be divided into work hours or leisure hours. The household faces the market hourly wage rate, W .

a. Write down the budget constraint of the household. (5pts)

$$C = w(T-L) + V$$

b. Define the indifference curve. (5pts)

The several combinations of C and L that gives the same level of utility.

c. Define the marginal rate of substitution between leisure and consumption using the marginal utility of leisure and consumption. Intuitively explain what this concept stands for. (5pts)

$MRS = \text{Marginal Utility of Leisure} / \text{Marginal Utility of Consumption}$

Intuitively speaking, this is the subjective value of leisure measured by the value of consumption.

d. What assumptions on utility function make the indifference curve convex to the origin? (5pts)

Marginal utility of consumption declines as consumption amount increases while the marginal utility of leisure declines as leisure time increases.

e. Suppose the optimal hours of leisure is an inner solution ($0 < L^* < T$), what is the condition that should hold at the optimal combination of (C, L)? (5pts)

$MRS = W$

f. Suppose a combination of (C, L) on the budget constraint does not satisfy the condition stated in e. Explain why the household can increase its utility by changing the combination of (C, L) satisfying the budget constraint. (5pts)

Suppose $MRS > W$ at a combination of C and L. In this situation, the subjective value of leisure is higher than the market price of leisure. By reducing W amount of consumption, the worker can enjoy an additional hour of leisure. The utility decreases by $MU_c * W$ due to the reduction of consumption but the utility increase by $MU_l * 1$ due to the increase in leisure hour. We know that $MU_l > MU_c * W$ because $MRS = MU_l / MU_c > W$. The same discussion can be made for the case $MRS < W$.

g. Suppose the wage rate, W, increases to W', ($W < W'$), does the optimal amount of consumption increase, decrease or you cannot tell? (5pts)

Due to the substitution effect, the demand for consumption increases because the relative price of consumption gets cheaper. The increase in wage rate increases income of the worker and, due to the income effect, the demand for consumption increases. Due to the both substitution and income effects, the optimal amount of consumption increases.

f. Suppose the utility function is given as

$$U = C^\alpha L^{1-\alpha}, 0 < \alpha < 1 .$$

Calculate the optimal hours of leisure as a function of W, T and V. (Hint, use the optimality condition obtained in e. and the budget constraint.) (10pts)

Optimality condition implies

$$MRS = MU_W / MU_C = (1-\alpha)C^\alpha L^{-\alpha} / \alpha C^{\alpha-1} L^{1-\alpha} = ((1-\alpha)/\alpha)(C/L) = W ,$$

From the condition above, we derive $C = LW(\alpha/(1-\alpha))$. We substitute this optimality condition into the budget constraint, $C = W(T - L) + V$.

$$LW \frac{\alpha}{1-\alpha} = W(T - L) + V$$

$$LW(1 + \frac{\alpha}{1-\alpha}) = WT + V$$

$$LW(\frac{1}{1-\alpha}) = WT + V$$

$$L^* = (1-\alpha) \frac{WT + V}{W} = (1-\alpha)(T + \frac{V}{W})$$

g. Using the result obtained in f., examine the effect of wage increase on the optimal hours of leisure. (5pts)

Increase in W decreases the optimal hours of leisure. Mathematically,

$$\frac{\partial L^*}{\partial W} = \frac{\partial(1-\alpha)(T + VW^{-1})}{\partial W} = -(1-\alpha)VW^{-2} < 0$$

2. Questions on Labor Demand (30pts)

a. Explain why the labor demand curve is downward sloping in the short run. (5pts)

Because of the law of diminishing returns.

b. Explain why the marginal product of labor curve and the average product of labor curve intersect at the maximum of the average product of labor curve. (5pts)

When marginal product is higher than average product, average product increase, on the other hand, when marginal product is lower than average product, average product decreases.

c. When wage rate exceeds a certain point, a firm shut down the operation in the short run because a firm run deficit by its operation. What is the upper bound wage rate below which a firm operates? (5pts)

When the price exceeds the price where the value of average product and the value of marginal product equate, the firm quits its operation. Above this price, even at the profit maximizing output quantity, the cost of production exceeds the revenue.

d. We now consider a firm's profit maximization behavior in the long run. What is the distinction between the short run and the long run? (5pts)

In the short run, the amount of capital input is fixed whereas in the long run, the amount of capital input is variable.

Suppose there is a cram school that produces education service using class rooms (capital) and teachers (labor). Suppose a combination of x_1 class rooms and x_2 teachers can produce education service for y pupils per hour and this combination of inputs cannot be altered (i.e. for example, if there are two teachers and a class room, this combination only can teach 15 pupils per hour. The same is true for the case of two blackboards and one teacher.). The relationship between x_1 , x_2 and y are tabulated as follows.

The number of class rooms (x_1)	The number of teachers (x_2)	The number of pupils taught per hour (y)
1	1	15
2	2	25
3	3	30
4	4	30
5	5	30

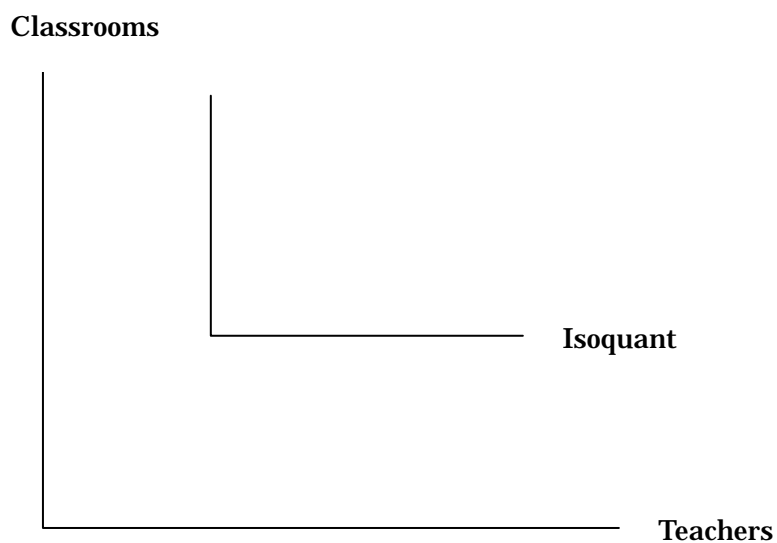
This cram school is a price taker and pay 10000 yen per hour for a teacher, 15000 yen to rent a room per hour and receive 2000 yen per hour from pupils as tuitions.

e. Under the above setting, what is the optimal number of class rooms and the optimal number of teachers? Explain your answer using the marginal cost-benefit framework. (5pts)

When the number of class rooms and the number of teachers both increase from 0 to 1, the marginal cost is $10000+15000=25000$. On contrary, the marginal revenue is $2000*15=30000$. Thus it is optimal to increase the number of class rooms and the number of teachers to 1 and 1. When both the classrooms and teachers increase from 1 to 2, the marginal cost is again 25000, whereas the marginal revenue is $(25-15)*2000=20000$. Because marginal cost exceeds the marginal revenue, it is optimal not to increase classrooms and teachers from 1 to 2. Thus the optimal combination of classrooms and teachers are both 1.

f. Suppose the wage rate of teachers increase to 15000 yen per hour. How many teachers are reduced due to the substitution effects? (Hint: Imagine the shape of isoquant curve for the production function given above.) (5pts.)

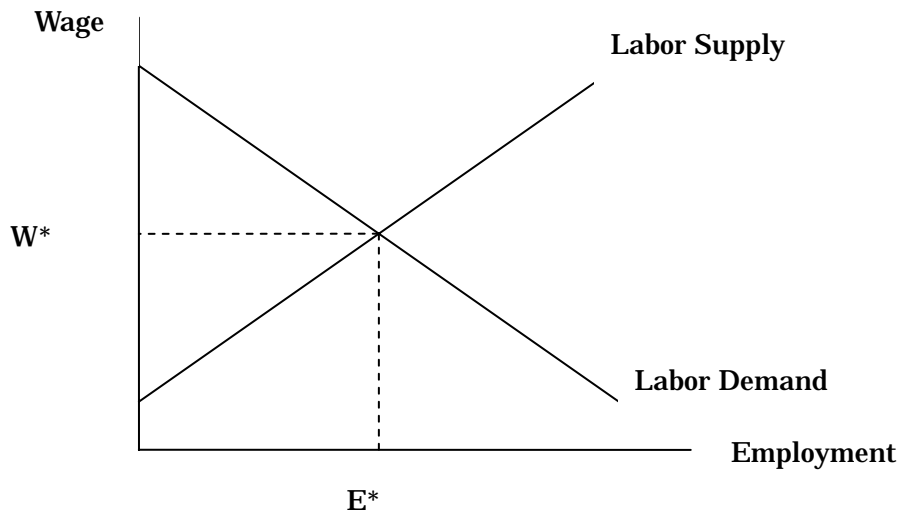
The isoquant curve looks like



Thus, even when the slope of isocost curve changes, as far as output is held constant, the optimal input of teachers does not change. Thus non teacher is reduced due to the substitution effect.

3. Questions on Market Equilibrium (20pts)

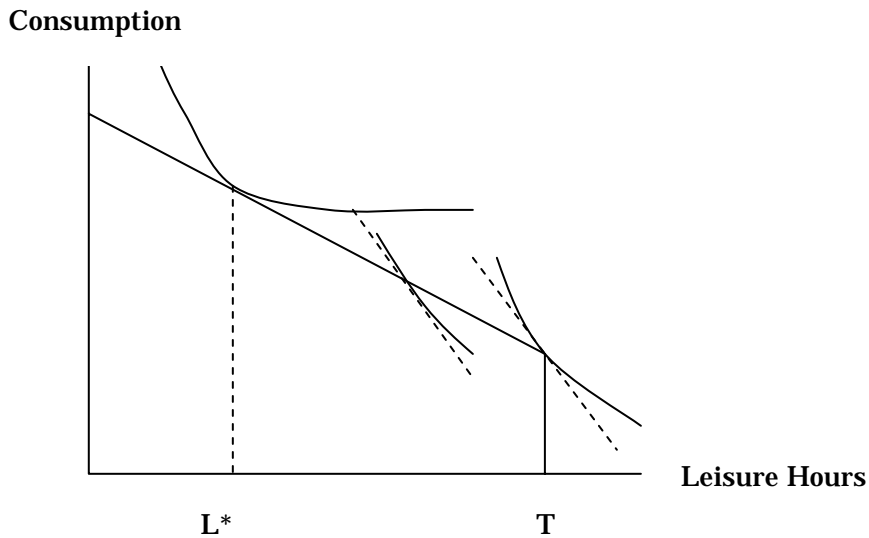
a. Typical labor market equilibrium is drawn as follows.



Explain why the area below equilibrium wage and above labor supply curve is considered as “workers’ surplus.” (5pts)

The labor supply curve corresponds to the value of time of workers. Thus if the value of time is below the wage rate (market price of time), the difference between the value of time and the market price of time is the gain from trade. This is the reason why the difference between labor supply curve and the wage rate is called as workers’ surplus.

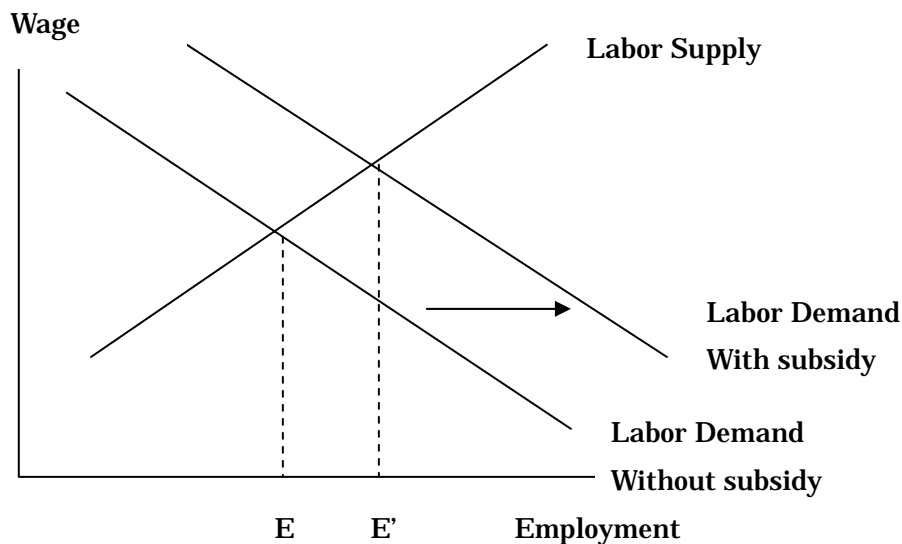
b. As we have seen in the class, we can derive the labor supply function from the household's utility maximization behavior. We used indifferent curves and a budget constraint to analyze the utility maximization behavior. In this familiar diagram, where does the worker's surplus show up? (Hint: What is the worker's surplus? It is the difference between what and what?) Explain your answer by drawing a diagram that put consumption on the vertical axis and leisure on the horizontal axis. (5pts)



The difference between the slopes of indifference curve (marginal rate of substitution = the value of time) and the budget constraint (wage rate = market price of time) is the gains from trade. This is the worker's surplus that appears at the individual level.

c. Innovation is considered to be an engine of economic growth. Based on this consensus, Japanese government spends a lot of money to subsidize the research and development activity by private firms and government research institutes. Suppose the number of innovations in Japan purely depends on the number of scientists working in Japan. Given this assumption, whether the subsidy policy promotes innovation or not depends on the labor supply elasticity of scientists. Explain why this is the case using a simple demand and supply framework. (10pts)

The subsidy increases the value of marginal product of scientist because the value of innovation increases due to the subsidy.



The increase of the number of scientists due to this subsidy policy depends on the slope of labor supply curve, which is basically labor supply elasticity.

Some of you pointed out the possibility that subsidy shifts labor supply curve down. This is also correct way of thinking. The increase in the number of scientists still depends on the slope of labor supply curve.