Homework 7

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Problem11.1

a

Mmerits of including $y_{i,t-1}$ in the model

The merits of including $y_{i,t-1}$ are two holds. First is to allow for state dependency; current state may depend on last period's state. Secondly the past level of unemployment may well work as a proxy variable to capture unobserved characteristics of a city.

Assumpution for pooled OLS

$\tilde{x}_{it} = [z_i, x_{it}, y_{it}, prog_{it}]$

Assumption1: $E[\tilde{x}'_{it}u_{it}]$ (orthogonality)

Assumption2: $E[\tilde{x}'_{it}\tilde{x}'_{it}]$ is nonsinglur.

b

Considering the equation with AR(1) serial correlations,

$$y_{it} = \theta_t + z_i\gamma_t + x_{it}\beta + \rho_1 y_{it-1} + \delta_1 prog_{it} + u_{it}$$

$$u_{it} = \zeta u_{it-1} + v_{it}$$

Then,

$$E[y_{it-1}u_{it}] = \zeta E[u_{it-1}^2].$$

As this shows, the orthgonality condition is violated, and thus pooled OLS estimators are inconsistent by the existence of u_{it}'s serial correlation

However, the fact doesn't necessary mean that the model is of limited value. The reason for this is that using IV renders consistent estimators of this model.

c

$$y_{it} = \theta_t + z_i\gamma + x_{it}\beta + \delta_1 prog_{it} + c_i + u_{it}$$

Under the strict exogenity assumption, $E[u_{it}|Z_i, X_i, prog_it] = 0$, the causal effect of training program on unemployment rate is consistently estimated.

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d

The model would be

$$y_{it} = \theta_t + z_i\gamma_t + x_{it}\beta + \rho_1 y_{it-1} + \delta_1 prog_{it} + c_i + u_{it}$$

Under the sequential exogenity assumption, $E[u_{it}|x_{it},,,,x_{i1},c_i]$, we can consistently estimate the model.

Firstly, taking first difference results in

$$\triangle y_{it} = \triangle\theta_t + \triangle x_{it}\beta + \rho_1\triangle y_{it-1} + \delta_1\triangle prog_{it} + \triangle u_{it},$$

and estimate the model by 2SLS or GMM using lagged y_{it} as instruments for y_{it-1}.

Problem11.3

$$y_{it} = \beta x_{it} + c_i + u_{it}$$

$$\ddot{y}_{it} = \beta\ddot{x}_{it} + \ddot{u}_{it}$$

$$\hat{\beta}_{FE} = \frac{\sum \ddot{x}_{it}\ddot{y}_{it}}{\sum \ddot{x}_{it}\ddot{x}_{it}}$$

plim $\hat{\beta}_{FE} = \frac{E(\ddot{x}_{it}\ddot{y}_{it})}{E(\ddot{x}_{it}y_{it})} = \frac{E((\ddot{x}^*_{it}+\ddot{r}_{it})\ddot{y}_{it})}{E((\ddot{x}^*_{it}+\ddot{r}_{it})(\ddot{x}^*_{it}+\ddot{r}_{it})}$

$$= \frac{E(\ddot{x}^*_{it} + \ddot{r}_{it})(\ddot{x}^*_{it}\beta + \ddot{u}_{it})}{E(\ddot{x}^*_{it} + \ddot{r}_{it})(\ddot{x}^*_{it} + \ddot{r}_{it})}$$

$$E[\ddot{x}^*_{it} + \ddot{r}_i]^2 = E[\ddot{x}^*_{it}]^2 + 2E[\ddot{x}^*_{it}\ddot{r}_i] + E[\ddot{r}_{it}]^2 = var(\ddot{x}^*_{it}) + var(\ddot{r}_{it})$$

$$E(\ddot{x}^*_{it} + \ddot{r}_{it})(\ddot{x}^*_{it}\beta + \ddot{u}_{it}) = \beta var(\ddot{x}^*_{it})$$

Thus,

plim$\hat{\beta}_{FE} = \frac{\beta var(\ddot{x}^*_{it})}{var(\ddot{x}^*_{it})+var(\ddot{r}_{it})} = \beta\frac{var(\ddot{x}^*_{it})+var(\ddot{r}_{it})-var(\ddot{r}_{it})}{var(\ddot{x}^*_{it})+var(\ddot{r}_{it})}$

$$= \beta(1 - \frac{var(\ddot{r}_{it})}{var(\ddot{x}^*_{it}) + var(\ddot{r}_{it})})$$

Problem11.4

a

$$y_{it} = \beta x_{it} + c_i + u_{it}$$

$$c_i = y_{it} - \beta x_{it} - u_{it}$$

$$\mu_c = Ec_i = Ey_{it} - \beta Ex_{it} - Eu_{it} = Ey_{it} - \beta Ex_{it}$$

By replacing population means and β with $\hat{\beta}_{FE}$,

$$\hat{\mu_c} = N^{-1} \sum_{i=1}^{N} T^{-1} \sum_{t=1}^{T} (y_{it} - \hat{\beta}_{FE} x_{it})$$

$$= N^{-1} \sum_{i=1}^{N} (\bar{y_i} - \hat{\beta}_{FE} \bar{x_i}).$$

b

$$y_{it} = \beta x_{it} + g_i t + c_i + u_{it}$$

First difference tranformation gives

$$\triangle y_{it} = \beta \triangle x_{it} + g_i + \triangle u_{it}$$

$$\mu_g = Eg_i = E\triangle y_{it} - \beta E\triangle x_{it} - E\triangle u_{it}$$

$$= E\triangle y_{it} - \beta E\triangle x_{it}$$

In the manner of Part A,

$$\hat{\mu_g} = N^{-1} \sum_{i=1}^{N} (\tilde{\triangle y_i} - \hat{\beta}_{FE} \tilde{\triangle x_i})$$

Problem11.16

a

It is possible to use FE estimation or FD estimation in order to estimate β, since the necessary assumptions for a consistent estimation, a strictly exogenity and a standard rank assumption, are guaranteed. Estimation by random effects, on the other hand, requires additional assumption; an unobserved heterogeneity and regressors should not be correlated.

b

$y_{it} = \gamma z_i + x_{it}\beta + c_i + u_{it}$
$E(y_{it}|z_i, x_i, w_i) = \gamma z_i + x_{it}\beta + E(c_i|z_i, x_i, w_i) + E(u_{it}|z_i, x_i, w_i)$ but since

$$E(c_i|z_i, x_i, w_i) = \delta_0 + \delta_1 w_i + \bar{x}_i\delta_2$$

and

$$E(u_{it}|z_i, x_i, w_i) = 0,$$

$E(y_{it}|z_i, x_i, w_i) = \gamma z_i + x_{it}\beta + \delta_0 + \delta_1 w_i + \bar{x}_i\delta_2$

c

The result of part b implies that γ is identified by a pooled OLS estimation. The estimation requires two assumptions for consistently. As mentioned at problem11.1, one of which is an orthogonality condition, the other is nonsiglarity of the regressors's second moment.

d

As going along with hint, the variance of error term is

$$j'j\sigma_a^2 + I\sigma_u^2$$

This error structure implies GLS random effects estimateion renders efficient estimaors.

Problem11.17

$\hat{a} = N^{-1}\sum_{i=1}^{N}(z_i'z_i)^{-1}z_i'(y_i - X_i\hat{\beta}_{FE})$
$\hat{a} - a = N^{-1}\sum_{i=1}^{N}(z_i'z_i)^{-1}z_i'(y_i - X_i\hat{\beta}_{FE}) - E[(z_i'z_i)^{-1}z_i'(y_i - X_i\beta)]$ but,

$$E[(z_i'z_i)^{-1}z_i'(y_i - X_i\beta)] = E[(z_i'z_i)^{-1}z_i'y_i] - E[(z_i'z_i)^{-1}z_i'X_i]\beta$$

$$= E[(z_i'z_i)^{-1}z_i'y_i] - E[(z_i'z_i)^{-1}z_i'X_i](\beta - \hat{\beta}_{FE} + \hat{\beta}_{FE})$$

$$= (E[(z_i'z_i)^{-1}z_i'y_i] - E[(z_i'z_i)^{-1}z_i'X_i]\hat{\beta}_{FE}) + E[(z_i'z_i)^{-1}z_i'X_i](\hat{\beta}_{FE} - \beta)$$

{using $(\hat{\beta}_{FE} - \beta) = [N^{-1}\sum \ddot{X}_{it}'\ddot{X}_{it}]^{-1}N^{-1}\sum \ddot{X}_{it}'\ddot{u}_{it} + o_p(2)$, }

$$= (E[(z_i'z_i)^{-1}z_i'y_i] - E[(z_i'z_i)^{-1}z_i'X_i]\hat{\beta}_{FE})$$

$$+E[(z_i'z_i)^{-1}z_i'X_i](N^{-1}\sum \ddot{X}_{it}'\ddot{X}_{it})]^{-1}N^{-1}\sum \ddot{X}_{it}'\ddot{u}_{it} + o_p(2)$$

Thus,

$$\hat{a}-a = N^{-1}\textstyle\sum_{i=1}^{N}(z_i'z_i)^{-1}z_i'(y_i-X_i\hat{\beta}_{FE})-(E[(z_i'z_i)^{-1}z_i'y_i]-E[(z_i'z_i)^{-1}z_i'X_i]\hat{\beta}_{FE})$$
$$-E[(z_i'z_i)^{-1}z_i'X_i]E(\ddot{X}_{it}'\ddot{X}_{it})]^{-1}N^{-1}\sum\ddot{X}_{it}'\ddot{u}_{it}+o_p(2)]$$

By replacing population moments with sample moments and neglecting $o_p(2)$ term, $\hat{a}\widehat{-}a = N^{-1}\sum_{i=1}^{N}\{(z_i'z_i)^{-1}z_i'(y_i - X_i\hat{\beta}_{FE})$

$$-(N^{-1}\sum[(z_i'z_i)^{-1}z_i'y_i] - N^{-1}\sum[(z_i'z_i)^{-1}z_i'X_i]\hat{\beta}_{FE})$$

$$-N^{-1}\sum[(z_i'z_i)^{-1}z_i'X_i]N^{-1}\sum(\ddot{X}_{it}'\ddot{X}_{it})]^{-1}\ddot{X}_{it}'\hat{u}_{it}\}$$

$$= N^{-1}\sum_{i=1}^{N}\{(\hat{s}_i - \hat{a}) - \hat{C}\hat{A}^{-1}\ddot{X}_i'\hat{u}_i\}$$

Because $\sqrt{N}(\hat{a}-a) \sim N[0, avar((\hat{a}_i - a_i)]$,

$$\widehat{avar}(\sqrt{N}(\hat{a}-a)) = N^{-1}\sum_{i=1}^{N}[(\hat{s}_i - \hat{a}) - \hat{C}\hat{A}^{-1}\ddot{X}_i'\hat{u}_i][(\hat{s}_i - \hat{a}) - \hat{C}\hat{A}^{-1}\ddot{X}_i'\hat{u}_i]'$$