Dual Poverty Trap: Intra- and Intergenerational Linkages in Frictional Labor Markets*

Ryo Horii† Masaru Sasaki‡

December 20, 2009

Abstract

This paper constructs an overlapping generations model with a frictional labor market to explain persistent low education in developing countries. When parents are uneducated, their children often face difficulties in finishing school and therefore are likely to remain uneducated. Moreover, if children expect that other children of the same generation will also not receive an education, they expect that firms will not create enough jobs for educated workers, and thus are further discouraged from schooling. These intergenerational and intragenerational mechanisms reinforce each other, creating a serious poverty trap. Escape from the trap requires the well-organized and combined implementation of a subsidy for schooling, support for disadvantaged children, and public awareness programs.

JEL Classification Numbers: O11, J62, J23.

Keywords: overlapping generations model, education, poverty trap, job search, coordination failure.

*The authors are grateful to Ryoichi Imai, Noritaka Kudoh, Kazuo Mino, Yasuhiro Sato, Takashi Shimizu, seminar participants at Doshisya University, Osaka University, and Kansai University, and conference attendees at the 2006 Japanese Economic Association Meeting at Osaka City University and the Public Economic Theory 2008 Seoul Meeting at Hangyang University for their valuable comments and suggestions. They also acknowledge the constructive remarks of the associate editor and two anonymous referees in improving the paper. This research is partially supported by the Japan Society for the Promotion of Science, Grant-in-Aid for Young Scientists No. 18730164. Any remaining errors are naturally ours.

†Correspondence: Graduate School of Economics and Management, Tohoku University, 27-1 Kawauchi, Aoba-ku, Sendai 980-8576, Japan.

‡Institute of Social and Economics Research, Osaka University, 6-1 Mihogaoka, Ibaraki, Osaka, 567-0047, JAPAN.
1 Introduction

In low income countries with a GNI per capita of $975 or less, the net school enrollment rate for secondary education was only 38.1% in 2007.\footnote{Source: Education Statistics (Edstats) Version 5.3, World Bank. This figure can be obtained through the DDP Quick Query on the World Bank’s website. The World Bank classifies countries as low income when their 2008 GNI per capita (calculated using its Atlas method) is $975 or less.} This low enrollment rate implies that the majority of the workforce is employed in less productive sectors and earns low incomes and this helps keep these countries in poverty. Why then has the enrollment rate remained so low? This paper develops a theory of persistent poverty and low education, focusing on both the transmission of educational attainment between generations and on the strategic complementarity in schooling choice within the same generation.

When the parents in a family are relatively uneducated, the home environment may be less favorable to study than otherwise, and in that case, children find it difficult to obtain an education. Recent microeconomic empirical studies strongly support the argument that parents’ (in particular, the mother’s) education status is one of the most important determinants of school enrollment and child labor, whereas the level of household wealth has a relatively minor effect (e.g., Kurosaki \textit{et al.} 2006, Bratti 2007).\footnote{See also Behrman and Rosenzweig (2002) and Antonovics and Goldberger (2005).} Thus, differential home environments generate an \textit{intergenerational} transmission mechanism through which low educational attainment is inherited from one generation to the next.

In contrast, frictions in labor markets can create a strategic complementarity in schooling choice among children of the same generation. To see this, suppose that the majority of other agents in the same generation do not receive an education. In such an economy, firms find it difficult to hire educated workers and so will not create many jobs that require education.\footnote{Empirical studies have found that secondary school enrollment is a statistically significant determinant of foreign direct investment (FDI) inflows (e.g., Noorbakhsh \textit{et al.} 2001). This finding is consistent with our viewpoint that, as in low-income countries, jobs that require skilled workers are} Even when some jobs are created, firms will simply fill
the position regardless of the worker’s educational attainments, rather than searching in vain for an educated worker at the risk of the job remaining vacant for a long time. On this basis, agents will be discouraged from seeking education because they can rationally anticipate that it will be difficult to find a high-paying job and that education will not make this process any easier. This suggests that a low enrollment rate may be a result of an intragenerational coordination failure, where children do not receive an education because others do not.

This paper develops a simple overlapping generations model that incorporates both the intergenerational transmission mechanism and the intragenerational coordination problem, and shows that the interaction of those two mechanisms can create a serious poverty trap. In fact, when only one of these mechanisms is present, the economy can escape poverty with little policy intervention. Even among uneducated parents, some will maintain a good family environment and therefore the rate of educational attainment will increase gradually over the generations. Further, even when the coordination failure problem is present, it does not rule out the possibility that children choose another good equilibrium. However, when these mechanisms are present simultaneously, they reinforce each other: low educational attainment in the previous generation necessarily causes coordination failure, and coordination failure makes the intergenerational mechanism more persistent. As a result, the economy is “dually” poverty trapped. An economy can be saved from this trap only by well-organized policy packages that simultaneously solve the intergenerational and the intragenerational problems. We show that an appropriate policy package generally comprises several stages, and each stage requires multiple policy instruments, including education subsidies, support for children in difficult home environments, and public awareness campaigns. Conversely,

often created by foreign firms in the form of FDI. See Section 6.1 for more discussion.

4 For instance, Wakabayashi (1998) found that poor employment opportunities discouraged parents from sending their children to secondary school based on interviews conducted in poor communities of the Roi-Et province in Thailand. Using a cross country analysis, Dulleck et al. (2006) shows that the take-up rates of education are lower in economies where new jobs are less likely to be created.
if the authority aims to solve only one of these problems, it will fail to resolve even partially the problem targeted. This could be one of the reasons why various forms of development assistance have been unsuccessful (see Easterly (2009) for a recent survey).

Our paper relates to a number of studies. To start with, many studies explain the low education poverty trap in terms of credit market imperfections (e.g., Galor and Zeira 1993; Maoz and Moav 1999; Moav 2002; Checchi and Garcia-Penalosa 2004). However, the limited success of student loan systems in low-income countries suggests that the credit market is not the only source of the problem. Complementing this body of work, our model shows the emergence of a persistent poverty trap under complete credit markets. A number of studies illustrate the possibility of poverty traps by combining the intergenerational transmission of educational attainment with parental fertility choice (e.g., Moav 2005; Lord and Rangazas 2006; Azarnert 2008). If educated parents have a comparative advantage in raising educated children (quality), and there is a trade-off between the quality and quantity of children, then the poor choose high fertility rates with low investment in child quality, causing a poverty trap. We instead combine the intergenerational transmission of educational attainment with a frictional labor market and derive a poverty trap under a constant fertility rate.

Many studies have also shown that coordination failure problems between workers and firms in the frictional labor market may generate multiple equilibria through thick market externalities (Laing, Palivos, and Wang 1995; Acemoglu 1997; Takii 1997; Dessy and Pallage 2001; Burdett and Smith 2002; see also Dulleck et al. 2006). While the low education equilibrium may be chosen by chance in these studies, our model explains why the economy necessarily falls into the low education equilibrium when

---

5 More precisely, Galor and Zeira (1993) showed that a poverty trap emerges when credit constraints combine with indivisibilities in human capital investment. Maoz and Moav (1999) and Checchi and Garcia-Penalosa (2004) introduce random shocks to allow for some intergenerational mobility. (We consider a similar extension in Section 6.2.) Moav (2002) showed that a poverty trap could emerge without nonconvexities in the technology, if the individual’s propensity to save increases with income. See Moav and Neeman (2008, Section 2) for additional references.
the intergenerational mechanism is present. In addition, we focus on the strategic
complementarity between children of the same generation rather than between workers
and firms. Finally, our study closely relates to the independent work by Kim (2009),
which combines the intergenerational and network externalities to derive a “Network
Trap.” In Kim (2009), the network externality is simply assumed, whereas in our
model the intragenerational coordination problem emerges endogenously from the job
creation decision and searching strategy of firms in the frictional labor market.

The remainder of the paper is organized as follows. Section 2 presents a simple
overlapping generations model and derives the schooling choices of children. Section 3
introduces a frictional labor market and derives the job creation decision and searching
strategy of firms. Section 4 investigates the equilibrium dynamics of the economy and
explains why a “dual” poverty trap emerges. In Section 5, we consider three types of
effective policy packages depending on the various situations. Section 6 discusses the
validity and robustness of our results. Section 7 concludes. The Appendix contains
the proofs of the propositions.

2 Behavior of Individuals

Consider an overlapping generations economy where each agent lives for two periods:
childhood and adulthood. In adulthood, agents can be either educated or uneducated,
depending on whether they received an education during their childhood. Each adult
agent has exactly one child, so that the number of agents in each generation is constant.
We normalize the number of agents in each generation to unity.

2.1 Childhood

The life of an agent born in period $t$ is as follows. Let $e_t$ denote the number (or
equivalently, fraction) of educated adult agents (parents) in period $t$. Then $e_t$ children
are born to educated parents, and $1 - e_t$ children are born to uneducated parents. In
the first period of their life, agents choose whether to attend school. If agents want to
become educated in adulthood, they must attend school and pay a certain amount of
effort. The amount of effort needed to finish schooling varies among agents, partly because of their own home environment. In particular, each child’s home environment depends on their parents’ educational attainment. When compared with educated parents, uneducated parents may be weaker in providing effective preschool training for their children, may be less concerned about maintaining a good environment for their children to study, and/or may be reluctant to send their children to school. Regardless, children of these parents find it difficult to obtain an education. We assume that among \((1 - e_t)\) agents born to uneducated parents, a fraction \(p\) \((0 < p < 1)\) faces such difficulties. This means that among the unit population of generation \(t\) children, \(p(1 - e_t)\) are “children in difficult environments” who face a very high effort cost of obtaining an education and therefore never receive an education. The remaining \(1 - p + pe_t\) agents in this generation, i.e., those born to educated parents plus the fraction \(1 - p\) of those born to uneducated parents, are “children in good environments” who can readily obtain an education if they attend school. For these children, foregone earnings are the only cost of schooling. Agents attending school will earn no income in their childhood, while those not attending school can do some simple work that pays \(z > 0\) units of goods.

### 2.2 Adulthood

In the second period of their life (period \(t+1\)), agents can work in either the modern or the traditional sector. In the modern sector, production takes place when a firm and a worker meet and agree to a division of output. Each pair of a firm and an educated worker produces \(\hat{y}_e\) units of goods, whereas each pair of a firm and an uneducated worker produces a smaller amount, \(\hat{y}_u < \hat{y}_e\).

The labor market in the modern sector is frictional and adult agents must search for

---

6 For simplicity, we assume education is indivisible. As discussed (see footnote 5), previous studies have shown that this indivisibility (i.e., nonconvexity in human capital investment) may generate a poverty trap when combined with credit market imperfections. This mechanism does not work in our model, where agents can borrow without constraints on credit.

7 We relax this assumption in Section 6.2 and introduce the differential costs of obtaining education.
a vacant job in a firm. Let us denote the probability of an educated worker successfully matching with a vacancy by $q_{t+1}$, and that for an uneducated worker by $q_{t+1} - \Delta q_{t+1}$. Both the matching probability for educated $q_{t+1} \in [0, 1]$ and the probability differential between educated and uneducated $\Delta q_{t+1} \in [0, q_{t+1}]$ are endogenous (to be explained in Section 3) but taken as given by each agent. If an adult agent fails to match with any vacancy in the modern sector, the agent works in the traditional sector where an employment position is easy to find with no friction. In that case, they earn $x < y_u$.

When an adult agent matches with a vacancy in a firm, the agent and the firm negotiate the division of the output. If the matched pair fails to agree on the division of output, the agent must then work in the traditional sector, where he/she can earn $x$, whereas the firm receives nothing from the vacant job. We assume that multiple contacts between agents and firms are not allowed in the same period. Given this, and assuming that both parties have equal bargaining power, the matched pair reaches the Nash bargaining solution as follows. When the matched agent is educated, the firm obtains $y_e \equiv (\hat{y}_e - x)/2$ and the educated agent’s income is $y_e + x$. If the matched agent is uneducated, the firm obtains $y_u \equiv (\hat{y}_u - x)/2$ and the uneducated agent’s income is $y_u + x$. We denote the income differential between educated and uneducated by $\Delta y \equiv y_e - y_u$.

### 2.3 Schooling Decision

Children in good environments choose to attend school if and only if it yields an expected lifetime income at least as high as what they would have obtained by not attending school. If agents in period $t$ decide to become educated, they must abandon their first-period income, and so their second period income is $y_e + x$ with probability $q_{t+1}$ and $x$ with probability $1 - q_{t+1}$. For simplicity, we assume that the economy considered is a small open economy, where the discount factor $\beta \in (0, 1)$ is constant and exogenously given.\(^8\) Then, the expected lifetime income is $\beta [q_{t+1} y_e + x]$, where $\beta \in (0, 1)$ denotes the discount factor. Similarly, the expected lifetime income when

---

\(^8\)We discuss the validity of this assumption in Section 6.1.
not attending school is \( z + \beta [(q_{t+1} - \Delta q_{t+1})y_u + x] \). They receive education if and only if the former is at least as high as the latter, or equivalently, \( \beta(q_{t+1}y_e - (q_{t+1} - \Delta q_{t+1})y_u) - z \geq 0 \).\(^9\)

Note that the agents born in period \( t \) decide whether to attend school in their childhood, while the probabilities of finding a job in period \( t + 1 \) (\( q_{t+1} \) and \( \Delta q_{t+1} \)) are yet to be determined. Therefore, their decision is based on their expectations of \( q_{t+1} \) and \( \Delta q_{t+1} \) as of period \( t \), which are denoted by \( E_t[q_{t+1}] \) and \( E_t[\Delta q_{t+1}] \). Using this, the above condition for education can be written in an intuitive way:

\[
E_t[q_{t+1}] \Delta y + E_t[\Delta q_{t+1}] y_u \geq z / \beta. \tag{1}
\]

The LHS of equation (1) shows that there are two possible benefits from receiving education. The first term represents the expected benefit from higher earnings conditional upon finding a job, whereas the second term represents the expected benefit from a higher possibility of finding a job. Children receive education if the sum of these is higher than the opportunity cost of education evaluated at time \( t + 1 \) (\( z / \beta \) on the RHS). In other words, if either the expected matching probability for educated or the expected differential in the matching probability is sufficiently high, it pays to receive education by giving up income in childhood.

The following summarizes the schooling decision. There are \( 1 - p + pe_t \) children in good environments who receive an education if and only if condition (1) holds. Children in difficult environments never receive an education. Therefore, the number of adult agents who become educated in period \( t + 1 \) is:

\[
e_{t+1} = \begin{cases} 
0 & \text{if } E_t[q_{t+1}] \Delta y + E_t[\Delta q_{t+1}] y_u < z / \beta, \\
1 - p + pe_t & \text{if } E_t[q_{t+1}] \Delta y + E_t[\Delta q_{t+1}] y_u \geq z / \beta. 
\end{cases} \tag{2}
\]

Equation (2) describes how the expected matching probabilities determine the number of educated workers supplied to the period \( t + 1 \) labor market. Given this, the following section examines how the actual matching probabilities are determined in the frictional labor market.

---

\(^9\)Agents are assumed to receive an education when they are indifferent in doing so.
3 Job Creation and Searching Strategy

In this economy, firms create vacant jobs through free entry with a constant cost of $k < y_u$, and these jobs, filled or not, deteriorate in one period. Once a vacancy is created, the firm searches for an appropriate worker in the frictional labor market. In this event, the firm chooses from two searching strategies: it may search for a match from both types of workers or search selectively from educated workers. In either case, the probability that a vacancy matches with a worker in period $t + 1$ is:

$$A \left( \frac{\ell_{t+1}}{v_{t+1}} \right)^\gamma,$$  

where $\gamma < 1$ and $A < (k/y_e)^{1-\gamma}$ are positive parameters of the matching technology,$^{10}$ $v_{t+1}$ is the number of (competing) vacancies searching for workers, and $\ell_{t+1}$ is the number of workers from which the firm searches for a match. Specifically, $\ell_{t+1} = 1$ if the firm searches from both types of workers and $\ell_{t+1} = e_{t+1}$ if it searches selectively from educated workers.$^{11}$

Throughout the paper, we focus on the symmetric equilibrium where all firms pursue the same searching strategy. Then, from (3), the number of matches in a period is:

$$M(v_{t+1}, \ell_{t+1}) \equiv v_{t+1} \cdot A \left( \frac{\ell_{t+1}}{v_{t+1}} \right)^\gamma = A\ell_{t+1}^{\gamma}v_{t+1}^{1-\gamma}. \quad (4)$$

Note that equation (4) is a standard Cobb–Douglas matching function with constant returns to scale.

Let us consider how many jobs are created though free entry. When a firm searches for a match from both types of workers, it matches with a worker with probability $A(1/v_{t+1})^\gamma$. Because firms are randomly matched with workers, the matched worker is uneducated with probability $1 - e_{t+1}$.\textsuperscript{12} Therefore, the expected revenue, conditional

---

$^{10}$We assume $A < (k/y_e)^{1-\gamma}$ so that the number of matches never exceeds the number of workers.

$^{11}$In our setting, firms never search only from uneducated workers because searching from both types will always give both a higher matching probability and a higher expected profit from the match.

$^{12}$Note that $1 - e_{t+1}$ is the proportion of uneducated workers among all adult workers. We assume that a vacancy can match at most once with a worker, and therefore the firm never rejects a matched worker even when he/she is uneducated.
Figure 1: Free-entry conditions

upon the vacant job being matched with a worker, is $e_{t+1}y_e + (1 - e_{t+1})y_u = e_{t+1}\Delta y + y_u$. Under this strategy, job creation occurs until the expected profit $A(1/v_{t+1})^\gamma (e_{t+1}\Delta y + y_u) - k$ becomes zero, or equivalently:

$$v_{t+1} = \left[ \frac{A(e_{t+1}\Delta y + y_u)}{k} \right]^{1/\gamma} \equiv V_B(e_{t+1}). \quad (5)$$

Next, consider the case in which a firm creates a vacancy and searches selectively from educated workers. The firm matches with a worker with a lower probability of $A(e_{t+1}/v_{t+1})^\gamma$, but, because the matched worker is necessarily educated, this match yields a higher revenue of $y_e$ to the firm. Job creation takes place until the expected profit $A(e_{t+1}/v_{t+1})^\gamma y_e - k$ becomes zero, or equivalently:

$$v_{t+1} = (Ay_e/k)^{1/\gamma}e_{t+1} \equiv V_S(e_{t+1}). \quad (6)$$

Figure 1 depicts the free-entry conditions for both searching strategies. Note that, if $v_{t+1} < \max\{V_B(e_{t+1}), V_S(e_{t+1})\}$, there is an opportunity for a new firm to obtain a positive expected profit by creating a new vacancy and then employing either of the searching strategies. Therefore, the number of vacancies in equilibrium must be determined at $v_{t+1} = \max\{V_B(e_{t+1}), V_S(e_{t+1})\}$. In equilibrium, all firms employ the searching strategy that corresponds to the higher of $V_B(e_{t+1})$ and $V_S(e_{t+1})$, because this is the only strategy that breaks even under this level of $v_{t+1}$. By comparing (5) and (6), we obtain the following result.
Proposition 1 Suppose that $e_{t+1} \in [0, 1)$. Then, the following applies.\textsuperscript{13}

(i) When $\gamma \geq \Delta y/y_e$, $V_B(e_{t+1}) > V_S(e_{t+1})$ always holds. In equilibrium, all firms search from both types of workers.

(ii) When $\gamma < \Delta y/y_e$, then there exists $\bar{e} \in (0, 1)$ such that $V_B(e) > V_S(e)$ holds if and only if $e < \bar{e}$. In equilibrium, all firms search from both types of workers if $e_{t+1} < \bar{e}$, and they search selectively from educated workers if $e_{t+1} \geq \bar{e}$.

Proof: in Appendix.

Note that, in the above proposition, threshold $\bar{e}$ is not defined for the case of $\gamma \geq \Delta y/y_e$. For completeness, let us suppose $\bar{e} \geq 1$ for $\gamma \geq \Delta y/y_e$. Then, Proposition 1 can be stated in a more compact way: all firms search from both types of workers if and only if $e_{t+1} < \bar{e}$. Intuitively, when $e_{t+1}$ is small, there is only a small chance of finding an educated worker, and firms choose to search from both types of workers. In addition, when $\gamma$ is larger than $\Delta y/y_e$, the probability of a firm finding an appropriate worker depends greatly on the number of potential candidates. In this case, it is always reasonable to search from both types of workers ($\bar{e} \geq 1$).

Given the amount of job creation and the searching strategy, we now obtain the matching probabilities for educated and uneducated workers. When $e_{t+1} < \bar{e}$ (including the case of $\gamma \geq \Delta y/y_e$), firms search from both types of workers. Then, the probability of a worker of any type matching with a vacant job is given by the total number of matches divided by the total number of workers: $q_{t+1} = M(V_B(e_{t+1}), 1)/1$ and $\Delta q_{t+1} = 0$. When $e_{t+1} \geq \bar{e}$, firms search selectively from educated workers. The probability of an educated worker matching with a vacant job is given by the total number of matches divided by the number of educated workers, i.e., $q_{t+1} = M(V_S(e_{t+1}), e_{t+1})/e_{t+1}$, and uneducated workers will never find a match, $\Delta q_{t+1} = -q_{t+1}$. 

\textsuperscript{13}For simplicity, we assume that firms search selectively from educated workers when the expected profits from the two strategies are the same. We also ignore the extreme case where all agents are educated ($e_{t+1} = 1$) because there is no distinction between the two strategies. In fact, as long as the economy starts with some uneducated agents ($e_0 < 1$), $e_{t+1} = 1$ is never reached in finite time.
and (6) into these expressions, we obtain:

\[
q_{t+1} = \begin{cases} 
m(e_{t+1} \Delta y + y_u)^{\alpha} & \text{if } e_{t+1} < \bar{e}, \\
my_e^\alpha & \text{if } e_{t+1} \geq \bar{e}
\end{cases}
\]

\[
\Delta q_{t+1} = \begin{cases} 
0 & \text{if } e_{t+1} < \bar{e}, \\
my_e^\alpha & \text{if } e_{t+1} \geq \bar{e}
\end{cases}
\]

where \(\alpha \equiv (1 - \gamma)/\gamma\) and \(m \equiv A^{1/\gamma}/k^\alpha\) are positive constants. Observe that the matching probability \(q_{t+1}\) is increasing in \(e_{t+1}\). If more children in period \(t\) receive an education, firms in period \(t + 1\) observe that a higher fraction of workers in the labor market are educated. As this raises the expected revenue from a matched pair, firms create more jobs in the modern sector, increasing the probability of workers finding a job. Moreover, when firms observe that there are sufficient educated workers, they search selectively from educated workers, creating a significant matching probability differential \(\Delta q_{t+1}\) between educated and uneducated workers.

4 Equilibrium

4.1 Rational Expectations Stage Equilibria

In Sections 2 and 3, we obtained a two-way relationship between the number of educated workers and the matching probabilities. In the schooling decision (2), given the number of educated parents in the previous generation \(e_t\), the expected matching probabilities \(E_t[q_{t+1}]\) and \(E_t[\Delta q_{t+1}]\) determine the number of children receiving education \(e_{t+1}\), who become educated workers in period \(t + 1\). Through job creation (7), the number of educated workers \(e_{t+1}\) in turn determines the actual matching probabilities \(q_{t+1}\) and \(\Delta q_{t+1}\). As long as agents are rational, the expected matching probabilities \(E_t[q_{t+1}]\) and \(E_t[\Delta q_{t+1}]\) must coincide with the actual probabilities \(q_{t+1}\) and \(\Delta q_{t+1}\), respectively. Therefore, under a given value of \(e_t\), the rational expectations (stage) equilibrium for this period is given by a triple of \(q_{t+1}\), \(\Delta q_{t+1}\), and \(e_{t+1}\) that simultaneously satisfy (2) and (7).
Let us first consider the simpler case of $\gamma \geq \Delta y/y_e$, in which firms always search from both types of workers and therefore $\Delta q_{t+1} = 0$ is known to all agents. Figure 2 depicts conditions (2) and (7) in $(e_{t+1}, q_{t+1})$ space, for small and large values of $e_t$. In this figure, the schooling locus represents the number of agents that choose to become educated $e_{t+1}$ as a function of expectation $E_t[q_{t+1}]$. The job creation locus depicts the probability of a worker finding a vacant job $q_{t+1}$ as a function of $e_{t+1}$. If these loci intersect, then it is a rational expectations stage equilibrium.

Panel (a) in Figure 2 illustrates the case where there are two rational expectations equilibria. If children in period $t$ expect the matching probability to be high, many of them receive education, firms will create many jobs in period $t + 1$, and the matching probability will actually be high. This verifies that the initial expectation is rational. We refer to this rational expectations equilibrium as the high education nonselective (HN) equilibrium. Conversely, if children expect the matching probability to be low, few children receive education, firms will create few jobs, and the matching probability is low. This is consistent with the initial expectation. We refer to this as the low education (L) equilibrium.

Observe that, as illustrated in panel (b) of Figure 2, the HN equilibrium does not exist when $e_t$ is low. Intuitively, when the great majority of parents in period $t$ is uneducated, the number of children in good environments is limited $(1 - p + pe_t$ is
small). In such a case, even when all of these children receive education, the number of educated workers in period $t+1$ will not be so large and the amount of job creation will be limited. Accordingly, the matching probability in period $t+1$ would not be sufficiently high to justify the opportunity cost of education. ($q_{t+1}$ would never reach $z/\beta \Delta y$). As children in period $t$ rationally expect this, they never receive an education. Thus, the L equilibrium is the only equilibrium possible.

Figure 3 illustrates the job creation and schooling loci for the case of $\gamma < \Delta y/y_e$. Recall that, from Proposition 1, firms search selectively from educated workers whenever there are more than $\tilde{e}$. Suppose there are sufficient educated parents so that the number of children in good environments is above $\tilde{e}$. If these children expect that firms will create enough jobs but employ only educated workers, they have a very strong incentive to receive an education. If more than $\tilde{e}$ children receive an education under this expectation, firms create many jobs and actually search selectively from educated workers, confirming the initial expectation. We call this equilibrium the high education selective (HS) equilibrium.

---

\[14\] If the number of children in good environments does not exceed $\tilde{e}$, firms always search from both types of workers on the basis of Proposition 1. Therefore, the analyses for the case $\gamma \geq \Delta y/y_e$ above apply whenever $1 - p + pe_t \leq \tilde{e}$.
To summarize, there are three types of equilibrium in this economy, and which type of equilibria actually exists depends on the number of educated parents $e_t$. Let us define the threshold values by:

$$\Phi \equiv (\bar{e} + p - 1)/p,$$  \hfill (8)

$$\Psi(z) \equiv \frac{1}{p\Delta y} \left[ \left( \frac{z}{\beta m \Delta y} \right)^{1/\alpha} - y_e + p\Delta y \right].$$ \hfill (9)

In (8), $\Phi$ represents the value of $e_t$ such that the number of children in good environments, $1 - p + pe_t$, coincides with $\bar{e}$. The HS equilibrium exists only when $e_t \geq \Phi$. In (9), $\Psi(z)$ is the value of $e_t$ such that the matching probability when all of the children in good environments receive education, $m((1 - p + pe_t)\Delta y + y_u)^\alpha$, coincides with $z/\beta \Delta y$. The HN equilibrium exists only when $e_t \geq \Psi(z)$. Using these, we can formally state the conditions under which each of the three types of equilibria exist.

**Proposition 2**  
 Given the number of educated parents $e_t$ in period $t$, the economy has the following rational expectations stage equilibria in period $t + 1$.

- **(L)** When $z \geq \beta m \Delta y y_u^\alpha$, a low education equilibrium exists, where $e_{t+1} = 0$, $q_{t+1} = my_u^\alpha$, $\Delta q_{t+1} = 0$.

- **(HN)** When $\Psi(z) \leq e_t < \Phi$, a high education nonselective equilibrium exists, where $e_{t+1} = 1 - p + pe_t$, $q_{t+1} = m((1 - p + pe_t)\Delta y + y_u)^\alpha$, $\Delta q_{t+1} = 0$.

- **(HS)** When $z \leq \beta my_e^{1+\alpha}$ and $e_t \geq \Phi$, a high education selective equilibrium exists, where $e_{t+1} = 1 - p + pe_t$, $q_{t+1} = my_e^\alpha$, $\Delta q_{t+1} = my_e^\alpha$.

**Proof in Appendix.**

Figure 4 graphically illustrates the result from Proposition 2. It depicts the set of stage equilibria against $e_t$ and $z$ for different values of $\gamma$ (recall that $\alpha \equiv (1 - \gamma)/\gamma$). Panel (a) shows the case of $\gamma \geq \Delta y/y_e$, where we have $\Phi \equiv (\bar{e} + p - 1)/p \geq 1$.\(^{15}\) This means that $e_t > \Phi$ never holds and thus only the HN and L equilibria

\(^{15}\)Note that from (8), $\Phi$ becomes 1 when $\bar{e} = 1$, and this occurs when $\gamma = \Delta y/y_e$ from Lemma 1 in Appendix. Similarly, $\Phi$ becomes 0 when $\bar{e} = 1 - p$. From Lemma 1, this means $F(1 - p) \equiv (1 - p)\Delta y + y_u - (1 - p)^\gamma y_e = 0$. Solving it for $\gamma$ gives $\gamma = \log(1 - p\Delta y/y_e)/\log(1 - p)$.
(a) $\gamma \geq \Delta y/y_e$  
($\Phi \geq 1$)

(b) $\frac{\log(1-p\Delta y/y_e)}{\log(1-p)} < \gamma < \Delta y/y_e$.  
($0 < \Phi < 1$)

Figure 4: Conditions for the existence of the three types of equilibria

are possible, depending on $e_t$ and $z$. Panel (b) shows the case where $\gamma$ is between $\log(1-p\Delta y/y_e)/\log(1-p)$ and $\Delta y/y_e$. In this case, $\Phi$ is between 0 and 1, which means that all three types of equilibria are possible, again depending on $e_t$ and $z$. Finally, if $\log(1-p\Delta y/y_e)/\log(1-p) \leq \gamma$, $\Phi$ becomes negative, and only the HS and L equilibria are possible (not depicted).
4.2 Dynamics and the Poverty Trap

In what follows, we consider the long-term dynamics of the economy in terms of the number of educated agents \(e_t\). In so doing, we make two assumptions about the parameters.

**Assumption 1**

(i) \(\gamma > \log(1 - p\Delta y/y_e)/\log(1 - p)\),

(ii) \(\beta m\Delta y(y_e - p\Delta y)\alpha < z < \beta m\Delta yy_e^\alpha\).

Property (i) rules out very small \(\gamma\) such that firms always search selectively from educated workers. Property (ii) assumes that the opportunity cost of education \(z\) is neither so small that the high education equilibrium always exists, nor so large that no one receives education. Mathematically, properties (i) and (ii) imply \(\Phi > 0\) and \(0 < \Psi(z) < 1\), respectively, and therefore jointly imply \(0 < \min\{\Phi, \Psi(z)\} < 1\). Under this assumption, we observe from Proposition 2 (and also Figure 4) that the economy has only the low education equilibrium if \(e_t\) is less than \(\min\{\Phi, \Psi(z)\}\), and it has both a low education equilibrium and a high education equilibrium (either HS or HN) for larger \(e_t\). Therefore, the number of educated agents evolves over generations according to:

\[
e_{t+1} = \begin{cases} 
0 & \text{if } e_t < \min\{\Phi, \Psi(z)\}, \\
\text{either 0 or } 1 - p + pe_t & \text{if } e_t \geq \min\{\Phi, \Psi(z)\}.
\end{cases}
\]

(10)

Figure 5 illustrates the pattern of dynamics of \(e_t\) implied by (10). We find that two steady state values of \(e_t\) exist. The first is a good steady state where all agents receive an education \((e_t = 1)\), find jobs with a high probability of \(q_{t+1} = my_e^\alpha\), and therefore aggregate income is high. In the second steady state, no agent receives an education \((e_t = 0)\), there are few prospects of finding a job \((q_{t+1} = my_u^\alpha)\), and so aggregate income is low. The economy converges to the good steady state if the number of educated agents in the initial adult generation, \(e_0\), is larger than the threshold level of \(\min\{\Phi, \Psi(z)\}\) and the high education equilibrium is chosen for all subsequent periods. However, if \(e_0\) is smaller than \(\min\{\Phi, \Psi(z)\}\), the only possibility is that the economy falls into a poverty trap. Formally, the following proposition follows from equation (10).
Figure 5: Dynamics and multiple steady states

**Proposition 3** Suppose that Assumption 1 holds. If \( e_0 \geq \min\{\Phi, \Psi(z)\} \), there is an equilibrium path that converges to \( e_t = 1 \). If \( e_0 < \min\{\Phi, \Psi(z)\} \), the only equilibrium path is such that \( e_t = 0 \) for all \( t \geq 1 \).

Proposition 3 clearly states that, under certain parameter values, the economy inevitably falls into a poverty trap if it starts with too few educated agents. Two underlying mechanisms jointly create this poverty trap. The first is an *intrigenerational* coordination failure. Note that, regardless of \( e_t \), the low education (L) equilibrium is always among the set of possible equilibria. Therefore, there is the possibility that children in some period \( t \) expect a low matching probability \( E_t[q_{t+1}] = my_u \), which corresponds to the L equilibrium in period \( t + 1 \). Given this expectation, they do not receive an education, and the L equilibrium is actually realized. Even when a high education equilibrium is among the set of possible equilibria, each agent cannot rationally change his/her expectation given the expectations of other agents. That is, if other agents expect a low matching probability, one can rationally expect that other agents will not receive an education and that, in the next period, firms observing this outcome will not create many jobs. Therefore, we cannot escape this bad outcome un-
less all children within the same generation coordinate their expectations to the high education equilibrium.\textsuperscript{16} The other mechanism is the \textit{intergenerational} linkage. When the number of educated parents $e_t$ is low, many children ($p(1 - e_t)$ of population 1) reside in difficult environments and therefore do not receive an education. Specifically, given the number of educated parents in period $t$ ($e_t$), the number of educated parents in period $t + 1$ ($e_{t+1}$) is bounded above by $1 - p + pe_t$, which is smaller when $e_t$ is smaller. Thus, the low educational attainment of one generation is partly inherited by the next generation.

Although both of the above mechanisms are important, each is not significant enough to individually explain the persistent poverty and low education found among low-income countries. The intragenerational coordination problem does not mean that agents necessarily fail to coordinate on the high education problem; the coordination failure is merely one possibility among multiple equilibria. Similarly, the problem of an intergenerational linkage does not by itself mean that low educational attainment is persistent; even among children born to uneducated parents, a fraction $1 - p$ are in (relatively) good environments. If they receive an education, the problem of low educational attainment will then be gradually resolved over succeeding generations.

However, when the intra- and intergenerational problems combine, they create a far more serious situation than what can be created by each alone. Suppose there are only a few educated parents (specifically, $e_t$ is below $\min\{\Phi, \Psi(z)\}$). This means that a significant fraction of children is in a difficult environment and therefore does not receive an education. Given this, agents can rationally expect that, in the next period, the fraction of educated workers will be low. Firms will then not create sufficient jobs and as a result the matching probability will be low. Therefore, even children

\textsuperscript{16}Earlier studies focused on the strategic complementarity between workers and firms because workers and firms determine their investment decisions at the same time (see Laing, Palivos, and Wang 1995; Takii 1997; Acemoglu 1997; and Burdett and Smith 2002). However, investment by firms, and therefore job creation, does not usually take as long as the time required by children to obtain an education. Therefore, we assume that firms choose to invest after they confirm the fraction of educated workers in the economy. In this case, a strategic complementarity emerges, not between firms and workers, rather among children.
in good environments do not receive an education. As a result, and as shown in Proposition 2, coordination on the high education equilibrium is impossible when $e_t < \min\{\Phi, \Psi(z)\}$. In this way, the intergenerational linkage of low education necessarily causes the intragenerational coordination failure.

Once an intragenerational coordination failure occurs, there are no educated parents in period $t + 1 (e_{t+1} = 0)$. That is, the intragenerational coordination failure fortifies the intergenerational linkage of low education ($e_t = 0$ means not only $e_{t+1} \leq 1 - p$ but also $e_{t+1} = 0$). This induces the majority of children in period $t + 1$ to choose to not receive an education, and again this causes the intragenerational coordination failure in generation $t + 1$, which is inherited by period $t + 2$, and so on. In this way, the intra- and intergenerational mechanisms interact with each other and perpetuate poverty and low education, as found in Proposition 3. We describe this situation as a dual poverty trap.

In the next section, we show that the interaction not only makes the poverty trap more persistent, but also makes it more difficult to escape from the trap.

5 Economic Policies

As explained in the last section, each of the intra- and intergenerational problems is not significant enough to individually explain the persistent poverty. A persistent poverty trap emerges only when both problems coexist. However, this does not mean that the economy in the dual poverty trap can be saved by removing only one of the problems. Consider an economy already in the dual poverty trap (i.e., $e_t < \min\{\Phi, \Psi(z)\}$). It is easy to see that the intragenerational coordination problem cannot be solved separately. Given that the majority of children are born to uneducated parents, the only rational expectation is that the matching probability in the next period is very low (recall Proposition 2). Any coordination device alone thus cannot change the expectations of children in such a situation.

Policies focusing only on the intergenerational problem are again unlikely to save
the trapped economy. As we explain in the following subsections, the provision of schooling subsidies and support for children in difficult environments will mitigate the intergenerational link of low education and make the high education equilibrium possible when they are implemented appropriately. However, the low education equilibrium is still a rational expectations equilibrium.\[17\] In a dually trapped economy, children in period $t$ know that the probability of finding a job in the modern sector has been low for a number of generations. Specifically, in the poverty trap, parents are mostly employed in the traditional sector and will inform their children directly based on their own experience that there is little chance of being employed in the modern sector. Given this, each child is likely to believe that other children in the same generation will not receive an education and that, similarly to today, the low education equilibrium will be realized in the next period.\[18\] As a result, they do not receive education.

As explained so far, any economic policy that focuses on only one aspect of the dual poverty trap is likely to fail. This suggests the necessity of the combined implementation of policies that simultaneously tackle the two problems. An example of this type of combination is the Female Secondary School Assistance Project in Bangladesh, jointly initiated by the Government of Bangladesh and the World Bank in 1993. In this project, stipends for female students attending school are combined with a female education awareness campaign, where videos and print materials are distributed nationally aiming “...to promote a supportive community environment for girls’ education through widespread awareness about the merits of female educational, economic, and social development.”\[19\] Similarly, the World Bank’s Africa division placed special

\[17\] Strictly speaking, the low education equilibrium disappears if all children receive quite a large schooling subsidy such that almost all of the foregone income is compensated. However, we ignore this possibility because such a policy is typically too costly to be implemented.

\[18\] Rostow (1991) refers to this phenomenon as “long run fatalism,” whereas Hoff and Pandey (2006) call it “social identities.” Chamley (2004) shows that a similar equilibrium will be repeatedly chosen, even if there is only a small uncertainty about the structure of the future economy.

\[19\] Each year the awareness campaign distributes about 340,800 brochures, 841,500 calendars, 300,000 stickers, and 3,250 diaries. Awards are also given and documentaries produced at a total cost of US$1.7 million dollars. See the World Bank’s website and search for project ID P009555.
emphasis on awareness campaigns in enhancing the education of girls in Mauritania and Guinea (see World Bank 2000), successfully raising the gross school enrollment rates of girls in Mauritania to 83.2% in 1997–98 from 39.3% in 1989–90, and in Guinea to 36.9% in 1997–98 from 21.7% in 1989–90.

In the following subsections, we examine the effects of subsidies and support programs in more detail when they are combined with similar public awareness programs.

5.1 Uniform Subsidy for Schooling

Let us consider a subsidy program in which all children that finish schooling in (generic) period $t$ receive subsidy $s_t$. Suppose that the economy is initially in the dual poverty trap: i.e., $\min\{\Phi, \Psi(z)\} > 0$ and $e_t = 0$. With this uniform subsidy program, the opportunity cost of education falls from $z$ to $z - s_t$. This means that $\Psi(z)$ falls to $\Psi(z - s_t)$ and the locus of the high education equilibrium extends to $\min\{\Phi, \Psi(z - s_t)\}$.

\footnote{For simplicity, we assume that the expenditure required for this program is covered by foreign aid and/or a nondistortionary tax on agents. A penalty on child labor, with the expected amount of the fine being $s_t$, works in the same way as the subsidy on schooling (see Basu 1999; Doepke and Zilibotti 2005). However, enforcement of a ban on child labor is generally difficult in low-income countries and may cost more than the subsidy program.}
Note that, from (9), \( e_t \geq \Psi(z - s_t) \iff s_t \geq z - \beta \Delta y m \left[ (1 - p + pe_t) \Delta y + y_u \right]^\alpha \equiv S(e_t). \) Therefore, by providing a uniform subsidy \( S(0) \equiv z - \beta \Delta y m (y_e - p \Delta y)^\alpha, \) the locus of the high education equilibrium extends to \( e_t = 0, \) as illustrated in Figure 6. This means that the high education equilibrium becomes one of the rational expectations equilibria at \( e_t = 0. \) By combining uniform subsidy \( S(0) \) with an awareness program to coordinate expectations, the high education equilibrium is realized and \( e_{t+1} \) increases to \( 1 - p \) (\( > e_t = 0 \)). As long as \( e_t < \min\{\Phi, \Psi(z)\}, \) the authority must continue to provide a uniform subsidy of at least \( S(e_t), \) as otherwise the high education equilibrium disappears and the economy reverts to the dual poverty trap. Once \( e_t \) reaches or exceeds \( \min\{\Phi, \Psi(z)\}, \) the economy will converge to the good steady state without further support.

To summarize, the uniform subsidy program is effective only when the amount of uniform subsidy is above a certain threshold and the program is continued for a certain time, both of which imply that the program requires a sizable sum of funds. Moreover, as \( S(e_t) \) is larger when \( e_t \) is smaller, the program typically requires the largest expenditure in the initial stages of the program. These issues raise some concern about the implementability of the program because the trapped economies (typically poor and less-developed countries) face difficulties in financing the required funds. Accordingly, they are forced to rely largely on foreign assistance; however, foreign assistance is again limited and the donor countries (or their constituencies) may be reluctant to bear a large burden in the initial stage before any intermediate results have been observed.

### 5.2 Targeted Subsidy and a Two-stage Approach

When funds are not sufficient to provide a uniform (broad-based) subsidy, a natural alternative is to target the subsidy on only a limited number of children. However, when the number of recipients is limited, the number of educated workers in the next period cannot be high, and thus the matching probabilities will be low, implying that the high education equilibrium will not be realized (here, we define the high education equilibrium as the equilibrium where the majority of children in good environments receive education). In such a case, the targeted children will receive education only if
the program compensates for most of the opportunity cost of schooling. Specifically, suppose that the authority chooses \( f_t \) children and provides them with a schooling subsidy (or scholarship) of \( z - \beta m \Delta y y_u^a \). This means that the opportunity cost of education for these children is now only \( \beta m \Delta y y_u^a \). By replacing \( z \) by \( \beta m \Delta y y_u^a \) in (1), we confirm that the recipients choose to receive education, even in the low education equilibrium, where \( E_t[q_{t+1}] = my_u^a \).\(^{21}\)

Note that, even though the amount of the targeted subsidy for each recipient, \( z - \beta m \Delta y y_u^a \), is larger than that for the uniform subsidy, \( S(0) \), the amount of initially required funds can be small when \( f_t \) is chosen appropriately. How can this program help the economy escape from the poverty trap in the long run? The dynamics of the economy now change from (10) to:

\[
e_{t+1} = \begin{cases} 
  f_t & \text{if } e_t < \min\{\Phi, \Psi(z - s_t)\}, \\
  \text{either } f_t \text{ or } 1 - p + pe_t & \text{if } e_t \geq \min\{\Phi, \Psi(z - s_t)\}.
\end{cases}
\]

Equation (11) implies that there is no point in continuing this program; \( e_t \) will remain at \( f_t \) as long as the economy is in the low education equilibrium. Nonetheless, the targeted subsidy program can be effective as a first stage in a combined policy package. Consider a policy package for a dually trapped economy, which provides targeted subsidies for \( f_t \) children in the first period and thereafter shifts to a combination of a uniform subsidy and awareness programs as explained in the previous subsection. In this package, the required amount of subsidy in the second stage is at most \( S(f_t) \), which is smaller than \( S(0) \). Intuitively, as there are already \( f_t \) educated parents, the locus of the high education equilibrium needs to extend only to \( f_t \), rather than to the origin.

This two-stage approach is particularly effective when \( \Psi(z) \) is significantly larger than \( \Phi \), as depicted in Figure 7. In this situation, the approach explained in the previous section will call for a large uniform subsidy. (Note that a large \( s_t \) is required

\(^{21}\)To be exact, the matching probability in this case is \( m(f_t \Delta y + y_u)^a \), which is slightly higher than \( my_u^a \) because \( f_t \) children are educated. Accordingly, the required amount of subsidy is slightly lower: \( z - \beta m \Delta y (f_t \Delta y + y_u)^a \), which is close to \( z - \beta m \Delta y y_u^a \) as long as \( f_t \) is small. We assume the authority chooses recipients from those in good educational environments. Therefore, \( f_t \) cannot exceed \( 1 - p + pe_t \).
to push down $\Psi(z - s_t)$ to the origin.) However, if the available funds make it possible to provide $\Phi$ children with a targeted subsidy, $e_t$ increases from zero to the threshold at $\Phi$. Then, in the second stage, we only need to make children aware that the high education selective equilibrium is now possible, where firms will search only among educated workers. This gives children a very strong incentive to receive an education because receiving education will now not only raise their expected income in the modern sector but also increase their chances of finding a good job.

### 5.3 Providing Education Support

We have so far focused on policies focusing on children in good environments because children in difficult environments are assumed to never receive an education. In fact, children in poor countries face various barriers to education depending on their family and social circumstances, and it is difficult for authorities to systematically solve these problems. Nonetheless, there are many nongovernment organizations (NGOs) around the world that seek to support these disadvantaged children through individual treatments. This subsection investigates how such efforts contribute to saving the economy from the poverty trap.
Suppose there are NGOs that provide educational opportunities with $a_t$ among $p(1 - e_t)$ children in difficult environments. Then, there are $1 - p + pe_t + a_t$ children in total that are willing to receive an education if doing so results in a higher lifetime income. Theoretically, this is the same situation as the case where there are $e_t + a_t/p$ educated parents in an economy without NGO support. (In that case, there are $1 - p + p(e_t + a_t/p)$ children in good environments.) Therefore, by replacing $e_t$ in (11) with $e_t + a_t/p$, we obtain the dynamics of the economy in the presence of education support:

$$e_{t+1} = \begin{cases} f_t & \text{if } e_t < \min\{\Phi, \Psi(z - s_t)\} - a_t/p, \\ \\
\text{either } f_t \text{ or } 1 - p + pe_t + a_t & \text{if } e_t \geq \min\{\Phi, \Psi(z - s_t)\} - a_t/p. \end{cases}$$ (12)

As illustrated in Figure 8, the introduction of education support shifts the locus of the high education equilibrium (and also the threshold level) to the left by the amount $a_t/p$. This suggests that support for children in difficult environments can even benefit children already in good environments by raising the possibility of coordinating expectations on the high education equilibrium.

In particular, when the threshold level of $e_t$ is given by $\Phi$ (i.e., when $\Psi(z) > \Phi$),

---

22We assume that $a_t$ is exogenous to the government because there is no systematic way to solve the problems of troubled families and therefore easily increase $a_t$.  

26
education support will play an important role in helping the economy escape from the poverty trap, because in such a situation a marginal increase in the uniform subsidy $s_t$ cannot lower the threshold $\min\{\Phi, \Psi(z - s_t)\} - a_t/p$. Moreover, when the education support is incorporated into the second stage of the two-stage approach in the previous subsection, the required number of $f_t$ in the first stage can be reduced from $\Phi$ to $\Phi - a_2/p$, where $a_2$ is the number of children in bad environments who receive education assistance in the second stage (see Figure 8). This reduces the financial burden required in the first stage and enhances the implementability of the policy package in poor developing countries.

6 Discussion

In this section, we discuss the validity and robustness of our results. The first subsection discusses the empirical validity of our setting regarding the return to education. In the second subsection, we relax the assumption that agents are homogenous in terms of their ability.

6.1 Number of Educated Workers and Return to Education

In our model, the intragenerational coordination problem and, hence, the multiple equilibria take place because the expected return to education is low when there are only a few educated workers; i.e., because the return to education is increasing with the number of educated workers, at least over some range. This property depends on our assumption of a small open economy, where firms (including foreign and multinational firms) can invest in creating new jobs as much as they wish under a given world interest rate. However, if the supply of physical capital is limited (as in a closed economy), the expected return to education will be decreasing with the number of educated workers: under a limited stock of capital, firms cannot create more jobs even when they find there are more educated workers, and as a result it becomes more difficult for an
educated worker to be matched with a job. In such a case, the equilibrium is always unique, and the dual poverty trap will not emerge.

Which of the above situations, an open or closed economy, is more appropriate in considering job creation in low income countries? In fact, and as far as investment in the modern sector is concerned, low-income countries are far from closed. Focusing on Sub-Saharan Africa, Ajayi (2006, Ch. 2) suggests that “Given the region’s low income and domestic savings level, its resource requirements and its limited ability to funds domestically, the bulk of its finance ... will have to come from abroad, mostly in the form of FDI.” In accordance with our model, there is evidence that an economy with more educated workers attracts more foreign investment. Noorbakhsh et al. (2001) empirically examine the geographical distribution of FDI, and find that secondary school enrollment (representing human capital) is a statistically significant and one of the most important determinants of FDI inflow. Blonigen et al. (2007) also show that FDI is likely to be attracted by countries with higher average schooling years.

When FDI is attracted to countries with more educated workers, the demand for skilled workers increases, which may raise the return to education. In fact, Feenstra and Hanson (1997) find empirical evidence in Mexico that the growth of FDI is correlated with the relative wages of skilled labor. Kijima (2006) found that the increase in wage inequality in urban India after the mid-1980s was mainly attributable to increases in the return to skills, and that the accelerating skill premium was because of the increase in the demand for skilled labor. Lin and Orazem (2003) report that the returns to skill in Taiwan have increased in the past two decades, despite a substantial increase in the supply of university graduates.

Another possible way to examine the validity of our setting is to compare the returns

---

23 Even in a closed economy, the rate of return to education can be positively correlated with the number of educated workers if the increased educational attainment leads to greater innovation and a skill-based technical change (SBTC). This, in turn, provides a greater incentive for children to receive an education. In the United States, there has been a substantial increase in educational attainment between 1940 and 2000. Restuccia and Vandenbroucke (2008) found that changes in relative earnings through SBTC can explain most of this increase in educational attainment.
to education between low and high income countries. Previously, it was considered that the average return to education in poor countries tended to be higher than in rich countries. Typical estimates for the returns to education in Sub-Saharan Africa were typically over 6%, with some well over 10% (e.g., Psacharopoulos 1994). However, recent studies show that these figures may result from omitted variable bias. Using a credible instrument and controlling for gender and the sector of residence, Uwaifo Oyelere (2010) found that the rate of return to education in 1990s Nigeria was only 2.8%. Consistent with our result, Uwaifo Oyelere (2010) suggests that the low average return to education explains the fall in the demand for education, the increased shift to rent-seeking activities and increased emigration rates from Nigeria during the 1990s.

6.2 Differences in Abilities

So far, we have assumed that the only differences between children is their home environment. This assumption has greatly simplified the analysis because all of the children in good environments receive an education in the high education equilibrium, whereas none receive an education in the low education equilibrium. However, in reality, children’s abilities and therefore their behavior are not the same, even among those in good home environments. Heterogeneity in ability creates endogenous intergenerational mobility and also dynamic evolutions in the income distribution. There is a literature on how these factors affect the possibility of economic development and changes in the development process (e.g., Galor and Tsiddon 1997; Maoz and Moav 1999; Checchi and Garcia-Penalosa 2004; Ikefuji and Horii 2007). Following these studies, we consider here a simple extension of our earlier model and examine how the results should be modified.

Suppose that, in addition to the opportunity cost \( z \), receiving education requires an extra cost of \( \varepsilon_{it} \) (or an extra benefit if \( \varepsilon_{it} \) is negative), which differs among children. One interpretation is that children have differences in their ability to learn. Alternatively, we may simply interpret \( z + \varepsilon_{it} \) as the opportunity cost of education, which differs among children for various reasons. The condition for receiving education (1) now becomes \( E_t[q_{t+1}]\Delta y + E_t[\Delta q_{t+1}]y_u \geq (z + \varepsilon_{it})/\beta \). Suppose that \( \varepsilon_{it} \) is independently,
normally distributed with mean 0 and standard deviation $\sigma > 0$. We maintain the assumption that children in bad environments never receive education. Then, instead of (2), the number of adult agents who become educated in period $t+1$ is:

$$e_{t+1} = (1 - p + pe_t) G [\beta (E_t[q_{t+1}]/\Delta y_{e} + E_t[\Delta q_{t+1}]y_u) - z],$$  

where $G(\cdot)$ represents the cumulative distribution function (CDF) of $\varepsilon_{it}$. The behavior of firms is the same as in Section 3.

Let us examine the equilibrium for a simpler case of $\gamma \geq \Delta y/y_e$, in which firms always search among both types of workers. Given $\Delta q_{t+1} = 0$, the number of educated agents in (13) becomes: $e_{t+1} = (1 - p + pe_t) G [\beta E_t[q_{t+1}]/\Delta y_{e} - z]$. Figure 9(a) depicts the stage equilibrium for this case. The schooling locus is now a smooth curve rather than a step function. As long as $\sigma$ is not so large and there are sufficient educated parents ($e_t$), the economy has both the low education (L) and high education nonselective (HN) equilibria, along with an “unstable” equilibrium in the middle. This is similar to the case of $\sigma = 0$ (see Figure 2(a)). However, observe that $e_{t+1} > 0$ in the L equilibrium and $e_{t+1} < 1 - p + pe_t$ in the HN equilibrium. This implies that some degree of social mobility always exists between educated and uneducated workers.

Figure 9(b) illustrates the typical dynamic mappings of the economy. Observe that,
when $\sigma$ is higher (i.e., with a mean-preserving spread in the abilities of children), the region of $e_t$ under which the high education equilibrium exists becomes narrower. To interpret this result, recall that a high education equilibrium is realized only when a certain mass of agents choose to receive education. Intuitively, when children are heterogeneous, they tend to behave differently from others, which makes it difficult for a certain mass of children to coordinate on the high education equilibrium. Accordingly, the amount of (uniform) subsidy required to save the economy from poverty is greater when the differences in abilities are wider. In the setting of Figure 9(b), it is necessary to compensate 9% of the opportunity cost of schooling ($s_0 = 0.09z$) in the first period of the uniform subsidy program of Section 5.1 if the abilities are the same ($\sigma = 0$). When children are heterogeneous and there is a 5% standard deviation in the schooling cost ($\sigma = 0.05z$), the required amount of initial subsidy increases to 16%, and increases further to 19% when $\sigma = 0.1z$.

Let us briefly discuss the case of $\gamma < \Delta y/y_e$. In this case, firms search selectively from educated workers if there are more than $\bar{e}$. As shown in (7), job creation for the case of $e_{t+1} \geq \bar{e}$ is characterized by $q_{t+1} = \Delta q_{t+1} = my_e^{\alpha}$. Substituting these into (13), we find that, if a high education selective (HS) equilibrium exists, the number of educated workers must be $e_{t+1} = (1-p+pe_t)G[\beta my_e^{1+\alpha} - z]$. The HS equilibrium exists if this value is actually larger than or equal to $\bar{e}$. It turns out that the HS equilibrium exists if and only if the number of educated parents satisfies:

$$e_t \geq \frac{1}{p} \left[ \frac{\bar{e}}{G[\beta my_e^{1+\alpha} - z]} + p - 1 \right].$$

(14)

It can be shown that the RHS in (14) coincides with $\Phi$ in (9) if $\sigma = 0$, and is increasing in $\sigma$ for $\sigma > 0$. With a mean spread in the ability of children, the region of $e_t$ under which the HS equilibrium exists shrinks. Firms are then less likely to search selectively from educated workers, and this undermines the incentive for receiving education and increases the possibility that the economy falls into a poverty trap. Accordingly, more

---

$^{24}$From Assumption 1(ii), $\beta my_e^{1+\alpha} > \beta my_e^{1+\alpha} > z$ holds. This means that, the expression in the CDF of the normal distribution $G[\cdot]$ is positive, and therefore the CDF is decreasing in standard deviation $\sigma$. In particular, the CDF is 1 if $\sigma = 0$, and it is below 1 if $\sigma > 0$. 

31
resources are needed to implement the policy packages discussed in Sections 5.2 and 5.3.

To summarize, when agents are heterogeneous in terms of learning ability, our model suggests that the economy is more likely to fall into a dual poverty trap. In addition, the policy package aiming to save the trapped economy will require more funds than in the case where abilities are homogeneous.

7 Conclusion

Using a simple overlapping generations model with a frictional labor market, this paper develops a theory of persistent poverty and low education by focusing on the interaction between the intergenerational transmission of educational attainment and the intragenerational coordination failure problem.

We show that the economy may have high and low education equilibria because of strategic complementarity between children of the same generation. When many children receive an education, firms can easily find educated workers. Firms will thus create many jobs and may search selectively from educated workers, which justifies children’s choice to receive an education. On the contrary, if only a few children receive an education, it becomes difficult for firms to find educated workers, not many jobs are created, firms do not search selectively from educated workers, and the children’s decision in not receiving an education becomes rational. In principle, which type of equilibrium is realized depends on expectations among children. However, the educational attainment of the previous generation may limit the type of expectation that can be held by the following generation. When the majority of parents in the economy are uneducated, even children in good environments cannot hold an optimistic expectation that firms will create many jobs, because they know that there are many disadvantaged children who will not receive an education. This may help explain the stagnant situation in many low-income countries where even nondisadvantaged children are reluctant to receive an education: as a result, low educational attainment is inherited from one generation by the next.
To save the economy from this poverty trap, it is necessary to make both the high education equilibrium possible and to make children believe that the high education equilibrium will actually take place. An appropriate policy package is to combine a uniform schooling subsidy program and an awareness program to convince children that their employment prospects are better than their parents’, although this package typically requires substantial funds at the initial stage. In some cases, a less costly alternative is to provide a targeted subsidy (or scholarships) for a limited number of children in the initial stage of the policy package. This may induce firms to later selectively search among educated workers, which in turn gives other nontargeted children a stronger incentive to receive an education. We also consider the effectiveness of an education support program for children in difficult environments and find that it is beneficial, even for those who do not receive support, because it can raise the possibility of coordinating their expectations on the high education equilibrium.

This paper analyzed the situation where two particular poverty trap mechanisms previously separately analyzed may reinforce each other. Obviously, there are many other reasons for persistent poverty (e.g., credit constraints, high fertility, insufficient capital accumulation, and so on), and low-income countries usual suffer from more than one of these problems, possibly in addition to the problems analyzed here. Our results suggest the necessity of a comprehensive policy package that should not be simply a collection of separate actions for each issue, but must be derived from a model that takes into account the interactions among the underlying issues.

Appendix

Proof of Proposition 1

From (5) and (6), it turns out $V_B(e) \geq V_S(e) \iff (V_B(e))^\gamma - (V_S(e))^\gamma \geq 0 \iff e \Delta y + y_u - e^\gamma y_e \geq 0$. The last expression has the following property:

**Lemma 1** Equation $F(e) \equiv e \Delta y + y_u - e^\gamma y_e = 0$ has generically two positive solutions. One is always $e = 1$, and the other solution, denoted by $\bar{e}$, is smaller than 1 if and only
if $\gamma < \Delta y / y_e$. In addition, given that $e \in [0, 1)$, $F(e) \geq 0$ holds if and only if $e \leq \bar{e}$.

**Proof of Lemma 1.** Let us consider the solution to $F(e) = e\Delta y + y_u - e^\gamma y_e = 0$ for $e \geq 0$. First, note that function $F(e)$ is strictly convex because $F''(e) = (1 - \gamma)\gamma e^{\gamma - 2}y_e > 0$ (recall that $\gamma \in (0, 1)$). In addition, $F(0) = y_u > 0$, $F(1) = \Delta y + y_u - y_e = 0$, and $\lim_{e \to \infty} F(e) = \lim_{e \to \infty} e^{\gamma(1-\gamma)\Delta y - y_e} + y_u = \infty$. These together imply that function $F(e)$ has a U-shape, the solution is at most 2, and one solution is at $e = 1$. Given these, whether equation $F(e) = 0$ has another solution than $e = 1$ depends on the slope of the function $F(e)$ at $e = 1$. When $\gamma < \Delta y / y_e$, Function $F(e)$ is upward sloping at $e = 1$ from $F'(1) = \Delta y - \gamma y_e > 0$, which implies that the other solution $\bar{e} \geq 0$ and $\bar{e} < e < 1$. Conversely, when $\gamma > \Delta y / y_e$, $F(e)$ is downward sloping at $e = 1$, implying that $\bar{e}$ is larger than 1 and that $F(e) \geq 0$ for all $e \in [0, 1)$. Finally, when $\gamma = \Delta y / y_e$, equation $F(e) = 0$ has repeated roots at $e = 1$ and $F(e) \geq 0$ always holds.

As $V_B(e) \geq V_S(e) \Leftrightarrow F(e) \geq 0$, Proposition 1 follows directly from Lemma 1.

**Proof of Proposition 2**

Here, we exhaustively derive all possible stage equilibria in the model in Section 4.1. Note that condition (7) suggests that the types of equilibria can be largely divided into equilibria with $e_{t+1} < \bar{e}$ and those with $e_{t+1} < \bar{e}$.

Let us first examine the possibility of $e_{t+1} < \bar{e}$, where (7) implies:

$$q_{t+1} = m(e_{t+1}\Delta y + y_u)^{\alpha}, \quad \Delta q_{t+1} = 0. \quad (15)$$

**(L)** Consider the case where children have an expectation $E_t[q_{t+1}] < z / \beta \Delta y$. Then, we have $e_{t+1} = 0$ from (2), which always conforms to the assumption of $e_{t+1} < \bar{e}$ given $\bar{e} > 0$. Note that $e_{t+1} = 0$ in (15) implies $q_{t+1} = my_u^\alpha$. Therefore, as long as $my_u^\alpha < z / \beta \Delta y$ or, equivalently, $z \geq \beta m \Delta y y_u^\alpha$, $E_t[q_{t+1}] = my_u^\alpha$ is in fact a rational expectation. This is the low-education (L) equilibrium.

**(HN)** Similarly, consider the case where children have an expectation $E_t[q_{t+1}] \geq z / \beta \Delta y$. Then, $e_{t+1} = 1 - p + pe_t$ children receive education from (2), which conforms
to assumption $e_{t+1} < \bar{e}$ if and only if $e_t < (\bar{e} + p - 1)/p \equiv \Phi$. From (15), the matching probability becomes $q_{t+1} = m((1 - p + pe_t)\Delta y + y_u)^\alpha$. Therefore as long as $m((1 - p + pe_t)\Delta y + y_u)^\alpha \geq z/\beta\Delta y$ or, equivalently, $e_t \geq \Psi(z)$, $E_t[q_{t+1}] = m((1 - p + pe_t)\Delta y + y_u)^\alpha$ is a rational expectation. This is the high-education nonselective (HN) equilibrium.

Next, let us examine the remaining possibility of $e_{t+1} \geq \bar{e}$, where (7) implies:

$$q_{t+1} = \Delta q_{t+1} = my_c^\alpha.$$ (16)

(HS) Note that for $e_{t+1}$ to be positive, (2) requires $E_t[q_{t+1}]\Delta y + E_t[\Delta q_{t+1}]y_u \geq z/\beta$. Using (16), this condition simplifies to $E_t[q_{t+1}] = E_t[\Delta q_{t+1}] = my_c^\alpha \geq z/\beta y_c$, or, equivalently, $z \leq \beta my_c^{1+\alpha}$. In this case, we have $e_{t+1} = 1 - p + pe_t$ from (2), which conforms to assumption $e_{t+1} \geq \bar{e}$ if and only if $e_t \geq (\bar{e} + p - 1)/p \equiv \Phi$.

This is the high-education selective (HS) equilibrium.

References


