Problem Set 10 on AS-AD model
(set on July 8th, due on July 15th)

I. A real wage function $G$ is modeled as $\frac{\alpha}{1 + \gamma} \left( \frac{Y}{Y^p} \right)^{\delta}$.

1. Derive $\pi_t = \pi_t^e + \theta \left( y_t - y_t^p \right)$ as an inflation version of AS curve.

2. Derive $\pi_t = \pi_t^e - \theta \left( u_t - u_t^p \right)$ as Phillips curve, where $u_t = \frac{U_t}{L} = \frac{L_t' - L_t}{L'}$, and $u_t^p = \frac{U_t^p}{L} = \frac{L_t' - L_t^p}{L'}$.

II. As described in my handout “A note on a simple version of new Keynesian model,” once nominal rigidity is taken into consideration under the assumption of rational expectations, an inflation version of AS curve is modified as $\pi_t = \pi_t^e + \theta \left( y_t - y_t^p \right)$. In this modified version, a permanent increase in monetary growth has no impact on current outputs, while its temporary increase raises current outputs to some extent. Describe in plain words, why implications of monetary policy for outputs depend on whether an increase in monetary growth is permanent or temporary.

III. Let us make a simple empirical exercise to explore how AS curves work in a real world. Of course, we are going to employ Japanese macroeconomic data. One serious problem associated with applications of AS curves to actual data is that it is impossible to directly observe either potential outputs or expected inflation. To overcome this problem in a somewhat casual manner, we employ adaptive expectations, that is, $\pi_t^e = \pi_{t-1}^e$, as a first approximation. As to computing potential outputs, we make a heroic approximation again, and assume that real outputs $Y_t^p$ grow at a constant rate $\mu$ or $Y_t^p = Y_0^p \left( 1 + \mu \right)^t$. Then, output gaps are computed by $\frac{Y_t - Y_t^p}{Y_t^p} \approx \ln Y_t - \ln Y_t^p \approx y_t - \left( y_t^p + \mu \times t \right)$.

As you learned in our class, real GNI is more appropriate in measuring aggregate outputs than real GDP. Hence, we employ the seasonally adjusted quarterly data of real GNI as real outputs and those of GNI deflators as price indexes. As shown in Figure 1, logarithmic outputs can be approximated by a linear equation $y_t^p + \mu \times t$, and
approximation errors may be regarded as output gaps \((y_i - y_i^p)\).

**Figure 1:**

The version of AS curve we are applying is \(\pi_t - \pi_{t-1} = \theta(y_i - y_i^p)\) where \(\theta > 0\). The left hand side of this equation (a change in inflation rates) is defined as a first difference of inflation from one year earlier, or \((p_t - p_{t-1}) - (p_{t-1} - p_{t-2})\), where \(p_t = \ln P_t\). Figure 2 plots the time series of both output gaps and changes in inflation rates.

In addition, Figures 3 through 5 are the scatter diagrams between both series for three sample periods.

**Figure 2:**

For the period between 1996 and 2003, output gaps and changes in inflation rates moved together, and the pattern is consistent with the AS curve with adaptive expectations (see Figure 3). However, this pattern broke down for the boom period between 2002 and 2007 (around six years with positive output gaps), and both series moved in opposite directions (Figure 4). There has been no more systematic relation between both series since 2008 (Figure 5).
As the last question of this lecture series, I would like to ask you why a standard shape of AS curves disappeared since the mid-2000s. I am sorry, but you have one more question to answer. Given these findings, is it still plausible to adopt mild inflation of 2 to 3 percent per year as a policy target?

HINT: Think as logically as possible! Do not make any illogical step in preparing for your answer. Recall from the classroom discussion how the slope of AS curves is determined, and which role the real wage function $G$ plays in the context of AS curves.

Figure 6 may be useful. Real GDP is approximately proportional to employment sizes, and real labor income is equal to real wages times employment sizes. Then, you may want to see how real wages changed with employment sizes.

### Figure 6

IV. Welcome any comment on my lecture in either English or Japanese!