Strategic Fragmentation and Production Subsidies*

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Abstract

In this paper, we examine the welfare properties of strategic fragmentation under production subsidies. We first consider a case in which a production subsidy rate is given exogenously. We show that, under the fixed subsidy policy, firms choose fragmentation despite the domestic production being socially desirable. Next, we examine a situation in which the government chooses a production subsidy rate optimally. In this case, we show that fragmentation is always socially desirable. This implies that the socially undesirable fragmentation does not arise under the optimal production subsidy policy.

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1 Introduction

In manufacturing sectors, production processes that used to be integrated within a country are increasingly disintegrated across countries. Feenstra (1998) provides empirical evidences that developed countries are outsourcing labor intensive production activities to low-wage countries. Such fragmentation of production significantly affects the nature of international trade. For instance, over the past decade, Japan experiences a significant increase in reverse imports from the rest of Asian countries.

The changing nature of international trade has caught attention of economists. The recent theoretical work on trade such as Venables (1999) and Jones (2000) analyze the effects of fragmentation on income distribution within a country. Some empirical studies such as Feenstra and Hanson (1996) show that outsourcing to low-wage countries help cause an increase in wage inequality between highly and less educated workers in developed countries. Those existing works mainly focus on the distributional effect of fragmentation of production processes. As a result, they use general equilibrium models with perfect competition.

Another important aspect of fragmentation is its impact on social welfare. Yomogida (2005) recently examines the welfare properties of fragmentation in the partial equilibrium model with imperfect competition. It shows that firms choose to produce overseas even though domestic production is desirable in terms of social welfare of firms’ home country. In order to prevent such socially undesirable fragmentation, the government may intervene in the firm’s choice of its production location. If the home government subsidizes domestic production by home firms, does it successfully prevent socially undesirable fragmentation from arising? In this paper, we examine this question.

We first examine a case in which a production subsidy rate is given exogenously. We show that socially undesirable fragmentation can still arise under a fixed production subsidy. Next, we consider a situation in which the home government chooses the subsidy rate to maximize its social welfare. Then, the optimal subsidy rate depends on the firm’s choice of its production location, i.e., the subsidy rate changes with the number of firms choosing
domestic production. In this case, we show that fragmentation is always socially desirable. In other words, under the optimal production subsidy, socially undesirable fragmentation does not arise.

The rest of this paper is organized as follows. In Section 2, we develop a model in which firms strategically choose the location of production. And we examine how a fixed subsidy affects the firms’ strategic choice of fragmentation. In Section 3, we turn to the case in which the government chooses a production subsidy rate optimally. We analyze the welfare properties of strategic fragmentation under the optimal production subsidy. In Section 4, we close this paper with a brief summary.

2 The model

Let us consider an economy with two countries, Home and Foreign. There is an industry in which two home firms produce a product, $X$. Each firm locates headquarters at Home, but it can choose the location of production. Whether it produces $X$ at Home or at Foreign depends on a difference in production costs. Since labor costs at Foreign are lower than those at Home, the firms may want to move its production to Foreign by FDI or outsourcing.\footnote{It is an important issue to examine whether firms choose outsourcing or FDI. In this paper, we focus on the firms’ choice of production locations, and thus we abstain from investigating the firms’ choice of organizational structures.} However, it would incur an additional cost to coordinate production activities at Foreign. Therefore, there is a trade-off between low labor costs and high coordination costs. We assume that home firms have the same production technology. Then, the cost function of firm $i$ is represented by

$$C(x_i) = \begin{cases} cx_i & \text{if } X \text{ is produced at Home}, \\ c^*x_i + f & \text{if } X \text{ is produced at Foreign}, \end{cases}$$

where $i = 1, 2$, $c$ and $c^*$ are marginal production costs at Home and Foreign, respectively, and $f$ is a constant fixed cost that represents a coordination cost. We assume that $c > c^* > 0$
and $f > 0$.

Now, we turn to the demand side. Suppose that product $X$ is consumed only at Home. Thus, if its production moves to Foreign, all outputs are exported back to Home. Preferences are quasi-linear and the demand for $X$ is represented by a linear demand function:

$$p = a - bD,$$

where $p$ is the price of good $X$, $D$ is its quantity of demand, and $a, b > 0$.

### 2.1 Fragmentation under the fixed subsidy

Given the demand and cost structure, we can derive the profit of each firm. Suppose that the home government provides a subsidy to the firm producing at Home. Let $s$ denote the subsidy per unit of output and $m$ denote the lump-sum subsidy. For a moment, we assume that $s$ and $m$ are given. Then, if firm $i$ produces at Home, then its profit is $[p - (c - s)]x_i + m$. On the other, if the firm moves its production to Foreign, then it must incur a transport cost to ship its outputs to the home market. Let $t$ denote the transport cost per unit of output. Then the profit of firm $i$ producing at Foreign is give by $[p - (c^* + t)]x_i - f$.

In the first stage, firms strategically choose the location of production, expecting the home government provides subsidies for domestic production. In the second stage, they choose their outputs in Cournot fashion. Solving the firm’s problem backward, the profit of each firm can be represented by the function of marginal costs. Let $x(\theta, \theta')$ denote the output of the firm with its own marginal cost $\theta$ and its rival’s marginal cost $\theta'$. It can be shown that

$$x \left( \theta, \theta' \right) = \frac{2(a - \theta) - (a - \theta')}{3b}.$$

Then, the profit of the firm, when its own marginal cost is $\theta$ and its rival’s marginal cost is $\theta'$, is represented as

$$\Pi(\theta, \theta') \equiv [p(x(\theta, \theta') + x(\theta', \theta)) - \theta] x(\theta, \theta').$$
Consider the following game:

\[
\begin{array}{|c|c|c|}
\hline
 & I & F \\
\hline
I & \Pi(c-s, c-s) + m, \Pi(c-s, c-s) + m & \Pi(c-s, c^* + t) + m, \Pi(c^* + t, c-s) - f \\
F & \Pi(c^* + t, c-s) - f, \Pi(c-s, c^* + t) + m & \Pi(c^* + t, c^* + t) - f, \Pi(c^* + t, c^* + t) - f \\
\hline
\end{array}
\]

Here, \(I\) denote integration and \(F\) denote fragmentation. If \(\Pi(c^* + t, c-s) - \Pi(c-s, c-s) > f + m\), then the firm would choose \(F\), given the rival’s choice of \(I\). Similarly, the firm would prefer \(F\) to \(I\) given the rival’s choice of \(F\) if \(\Pi(c^* + t, c^* + t) - \Pi(c-s, c^* + t) > f + m\). Figure 1 shows the Nash equilibrium of the firms’ choice of a production location. Since a reduction in the transport cost makes fragmentation more profitable, \(y = \Pi(c^* + t, c-s) - \Pi(c-s, c-s)\) and \(y = \Pi(c^* + t, c^* + t) - \Pi(c-s, c^* + t)\) are drawn as downward sloping curves in Figure 1. Notice that for each \(t \in [0, c-s - c^*)\), we have \(\Pi(c^* + t, c-s) - \Pi(c-s, c-s) > \Pi(c^* + t, c^* + t) - \Pi(c-s, c^* + t)\). This means that it is more profitable to switch to \(F\) from \(I\) if the rival still chooses integration than if it already chooses overseas production. Suppose that \(f + m\) is greater than \(\Pi(c^*, c-s) - \Pi(c-s, c-s)\). Then, it is never profitable for any firms to produce \(X\) overseas. Thus, both firms would choose integration at a Nash equilibrium. If \(f + m\) is smaller than \(\Pi(c^*, c-s) - \Pi(c-s, c-s)\), then firms’ optimal strategies depend on \(t\) as well as \(f + m\).

As in Figure 1, if \(t < c-s - c^*\), then the Nash equilibrium choices of firms’ production locations depend on \(t\) and \(f + m\) in the following manner:

1. If \(f + m > \Pi(c-s, c-s) - \Pi(c^* + t, c-s)\), then both firms optimally choose to produce \(X\) at Home;

2. If \(\Pi(c^* + t, c^* + t) - \Pi(c-s, c^* + t) < f + m < \Pi(c-s, c-s) - \Pi(c^* + t, c-s)\), then one firm chooses to move the production of \(X\) overseas and the other firm chooses to integrate the production of \(X\) at Home;

3. If \(f + m < \Pi(c^* + t, c^* + t) - \Pi(c-s, c^* + t)\), then both firms choose fragmentation and produce \(X\) overseas.
2.2 Welfare properties of fragmentation

It is possible to derive the social welfare of Home as a function of marginal costs. For each pair \( \theta, \theta' \in \{c - s, c^* + t\} \), let

\[
CS(\theta, \theta') \equiv \frac{(2a - \theta - \theta')^2}{18b},
\]

\[
W(\theta, \theta') \equiv CS(\theta, \theta') + \Pi(\theta, \theta') + \Pi(\theta', \theta).
\]

If firms choose to integrate the production process at Home, then the welfare of Home is

\[
W(c - s, c - s) - 2sx(c - s, c - s).
\]

Similarly, if one firm chooses fragmentation and the other still produces at Home, then the social welfare of Home is

\[
W(c^* + t, c - s) - sx(c - s, c^* + t) - f.
\]

If both firms choose fragmentation, then the social welfare of Home is

\[
W(c^* + t, c^* + t) - 2f.
\]

A change in the home welfare caused by fragmentation crucially depends on the transport cost and the coordination cost. If

\[
W(c^* + t, c - s) - W(c - s, c - s) + sx(c - s, c^* + t) > f,
\]

then the fragmentation raises the social welfare of Home.\(^2\) Similarly, the fragmentation of both firms benefits Home if

\[
W(c^* + t, c^* + t) - W(c^* + t, c - s) + sx(c - s, c^* + t) > f.
\]

Figure 2 shows a change in Home welfare caused by fragmentation. Since a reduction in the transport cost increases gains from fragmentation for Home, \( y = W(c^* + t, c - s) - W(c - s, c - s) + sx(c^* + t, c^* + t) \) and \( y = W(c^* + t, c^* + t) - W(c^* + t, c - s) + sx(c - s, c^* + t) \) are drawn as downward sloping curves. Observe that for each \( t \in [0, c - s - c^*] \), we have

\[
W(c^* + t, c - s) - W(c - s, c - s) + sx(c - s, c^* + t) > W(c^* + t, c^* + t) - W(c^* + t, c - s) + sx(c - s, c^* + t).
\]

This implies that gains from fragmentation for Home is larger when only one firm producing abroad than those when both firms moving their production overseas.\(^3\)

The choice of the production location is not necessarily socially desirable. Suppose that \( m = 0 \): the lump-sum subsidy is zero. In this case, it is easy to show that there are two

\(^2\)Note that \( 2sx(c - s, c - s) - sx(c - s, c^* + t) = sx(c^* + t, c^* + t) \).

\(^3\)A switch in the production regime from \((F, I)\) to \((F, F)\) provides a larger increase in consumer surplus than that from \((I, I)\) to \((F, I)\) because a reduction in the price is greater and the level of output is larger in the former than in the latter. Contrary to this, the negative effect of fragmentation on the profit of the firm remaining at home is greater if the production regime shifts from \((F, I)\) to \((F, F)\) than if it does from \((I, I)\) to \((F, I)\). The overall effect of fragmentation on \( W \) depends on the relative size of these two effects. See Appendix for the proof.
types of production regimes that are socially undesirable. First, suppose that \((t, f)\) satisfies

\[
f > \max \{ \Pi(c^* + t, c - s) - \Pi(c - s, c - s), W(c^* + t, c^* + t) - W(c^* + t, c - s) + sx(c - s, c^* + t) \}
\]

and

\[
f < W(c^* + t, c - s) - W(c - s, c - s) + sx(c^* + t, c^* + t).
\]

Then, \((F, I)\) is a socially desirable production regime, but both firms choose \(I\). On the other hand, there is another type of the equilibrium production regime that is socially undesirable. Suppose that \((t, f)\) satisfies

\[
f > W(c^* + t, c^* + t) - W(c - s, c^* + t) + sx(c - s, c^* + t), \text{ and}
\]

\[
f < \Pi(c^* + t, c^* + t) - \Pi(c - s, c^* + t).
\]

Then, \((F, I)\) is socially desirable but both firms choose \(F\). This suggests that domestic production is socially desirable but both firms choose to move their production overseas.

### 3 Optimal production subsidy

In this section, we consider a situation in which the home government chooses the optimal production subsidy rate. We assume that the government chooses the subsidy rate \(s\) to maximize the welfare of Home. The optimal subsidy rate depends on the production regime. If both firms choose to produce at home, then the home government would choose the subsidy rate \(s\) to maximize

\[
W(c - s, c - s) - 2sx(c - s, c - s).
\]

It can be shown that the optimal subsidy rate is

\[
s = \frac{a - c}{2}.
\]
If only one firm produces domestically and the other one chooses fragmentation, then the objective function for the home government is

$$W(c - s, c^* + t) - sx(c - s, c^* + t).$$

Then, we can show that the optimal subsidy is

$$\hat{s} = (a - c) - 4(c - c^* - t)$$

Notice that the optimal subsidy rate $\hat{s}$ may be negative if the transport cost is very small. In fact, the government would levy the optimal production tax on the firm producing at home if $t < c - c^* - \frac{a - c}{4}$.\(^4\)

Given the optimal production subsidy rate, firms strategically choose the location production. After the choice of the production location, firms compete in Cournot fashion. Then, the game can be represented as follows:

<table>
<thead>
<tr>
<th></th>
<th>$I$</th>
<th>$F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I$</td>
<td>$\Pi(c - \bar{s}, c - \bar{s})$, $\Pi(c - \bar{s}, c - \bar{s})$</td>
<td>$\Pi(c - \hat{s}, c^* + t)$, $\Pi(c^* + t, c - \hat{s}) - f$</td>
</tr>
<tr>
<td>$F$</td>
<td>$\Pi(c^* + t, c - \hat{s}) - f$, $\Pi(c - \hat{s}, c^* + t)$</td>
<td>$\Pi(c^* + t, c^* + t) - f$, $\Pi(c^* + t, c^* + t) - f$</td>
</tr>
</tbody>
</table>

Note that the lump-sum subsidy $m$ is assumed to be zero. Given the rival chooses $I$, it is optimal for the firm to shift its production overseas if $\Pi(c^* + t, c - \hat{s}) - \Pi(c - \bar{s}, c - \bar{s}) > f$. It can be shown that

$$\Pi(c^* + t, c - \hat{s}) - \Pi(c - \bar{s}, c - \bar{s}) = \frac{4[c - c^* - t]^2}{b} - \frac{(a - c)^2}{4b}.$$ 

Thus, it is easy to show that $\Pi(c^* + t, c - \hat{s}) - \Pi(c - \bar{s}, c - \bar{s})$ is decreasing with $t$. For the firm, gains from fragmentation is not necessarily positive. We can easily show that $\Pi(c^* + t, c - \hat{s}) - \Pi(c - \bar{s}, c - \bar{s}) > 0$ if $t < c - c^* - \frac{a - c}{4}$. Recall that $t < c - c^* - \frac{a - c}{4}$ if and only

\(^4\)The firm can obtain positive profits from domestic production even in the case of the optimal production tax. The output of the firm producing domestically is given by $x(c - \hat{s}, c^* + t) = \frac{a - c - b(c - (c^* + t))}{b}$, which is positive if $t > c - c^* - \frac{a - c}{3}$. This implies that, if $t \in (c - c^* - \frac{a - c}{3}, c - c^* - \frac{a - c}{4})$, then the output of the firm producing domestically $x(c - \hat{s}, c^* + t)$ is positive under the optimal production tax $\hat{s}$. 

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if \( \hat{s} = (a - c) - 4(c - c^* - t) < 0 \). That is, it is profitable for the firm to choose fragmentation if the optimal tax is levied on the rival producing domestically. Similarly, for the given choice of \( F \) by the rival, the firm would switch to \( F \) from \( I \) if \( \Pi(c^* + t, c^* + t) - \Pi(c - \hat{s}, c^* + t) > f \).

We can show that

\[
\Pi(c^* + t, c^* + t) - \Pi(c - \hat{s}, c^* + t) = \left[ a - c^* - t \right]^2 - \frac{[a - c - 3(c - c^* - t)]^2}{9b}. 
\]

This implies that \( \Pi(c^* + t, c^* + t) - \Pi(c - \hat{s}, c^* + t) \) is decreasing with \( t \). It is also easy to check that \( \Pi(c^* + t, c^* + t) - \Pi(c - \hat{s}, c^* + t) > 0 \) if \( t < c - c^* - \frac{a - c}{3} \).

Figure 3 shows Nash equilibrium in the choice of production location by the firm. The transport cost must be larger than \( c - c^* - \frac{a - c}{3} \). This is because \( x(c - \hat{s}, c^* + t) = \frac{a - c - 3(c - (c^* + t))}{b} \), which is positive if \( t > c - c^* - \frac{a - c}{3} \). That is, for the given choice of fragmentation by the rival, the output of the firm producing domestically can be positive if \( t > c - c^* - \frac{a - c}{3} \). It can be shown that, for all \( t \in [c - c^* - \frac{a - c}{3}, c - c^* - \frac{a - c}{5}] \),

\[
\Pi(c^* + t, c^* + t) - \Pi(c - \hat{s}, c^* + t) > \Pi(c^* + t, c - \hat{s}) - \Pi(c - \hat{s}, c - \hat{s}).
\]

This inequality implies that an increase in the firm’s gross profit by switching to fragmentation is greater when the rival chooses fragmentation than that when the rival chooses domestic production. Thus, the equilibrium choice of production location depends on the transport cost and coordination cost in the following manner.

**Proposition 1** Suppose that the home government chooses the optimal production subsidy. Then,

1. If \( f > \Pi(c^* + t, c^* + t) - \Pi(c - \hat{s}, c^* + t) \), then both firms choose domestic production.
2. If \( \Pi(c^* + t, c^* + t) - \Pi(c - \hat{s}, c^* + t) > f > \Pi(c^* + t, c - \hat{s}) - \Pi(c - \hat{s}, c - \hat{s}) \), then both firms choose domestic production or both firms choose fragmentation.
3. If \( f < \Pi(c^* + t, c - \hat{s}) - \Pi(c - \hat{s}, c - \hat{s}) \), then both firms choose fragmentation.
Recall that there exists an equilibrium in which only one firm chooses fragmentation in the case of fixed production subsidy. When the home government chooses the subsidy optimally, then the asymmetric production regime does not arise in equilibrium. Thus, a reduction in transport costs would induce the simultaneous choice of fragmentation by the firms.

3.1 Welfare properties of fragmentation under the optimal subsidy

Finally, we can easily check whether or not the equilibrium choice of production location is socially desirable. We can show that, if both firms choose domestic production, the welfare of Home is given by

\[ W(c - \bar{s}, c - \bar{s}) = \frac{(a - c)^2}{2b}. \]

Similarly, if both firms choose fragmentation, then the Home welfare is

\[ W(c^* + t, c^* + t) = \frac{4(a - c^* - t)^2}{9b}. \]

We can show that \( W(c^* + t, c^* + t) - W(c - \bar{s}, c - \bar{s}) > 0 \) if

\[ t < c - c^* - \frac{3 - \sqrt{8}}{\sqrt{8}}(a - c). \]

Note that \( c - c^* - \frac{3 - \sqrt{8}}{\sqrt{8}}(a - c) > c - c^* - \frac{a-c}{3} \). It can be shown that, for all \( t \in [c - c^* - \frac{a-c}{5}, c - c^* - \frac{a-c}{3}] \),

\[ W(c^* + t, c^* + t) - W(c - \bar{s}, c - \bar{s}) > \Pi(c^* + t, c^* + t) - \Pi(c - \tilde{s}, c^* + t). \]

Thus, if \( W(c^* + t, c^* + t) - W(c - \bar{s}, c - \bar{s}) > f > \Pi(c^* + t, c^* + t) - \Pi(c - \tilde{s}, c^* + t) \), then both firms choose domestic production despite that fragmentation by both firms is more socially desirable. If \( \Pi(c^* + t, c^* + t) - \Pi(c - \tilde{s}, c^* + t) > f > \Pi(c^* + t, c - \tilde{s}) - \Pi(c - \bar{s}, c - \bar{s}) \),
then both firms may choose to produce domestically. Then, the production regime is not desirable from the viewpoint of Home welfare. However, if both firms choose fragmentation, then the equilibrium choice of production location is socially desirable. Finally, if $f < \Pi(c^* + t, c - s) - \Pi(c - s, c^* - s)$, then either firm chooses to move the production overseas, and the fragmentation is socially more desirable than domestic production.

When the government chooses the optimal production subsidy, there is a possibility in which firms choose domestic production even though fragmentation is more desirable from the viewpoint of Home welfare. However, in contrast to the case of the fixed subsidy, there does not exist the possibility in which firms choose fragmentation even if domestic production is socially more desirable than fragmentation.

**Proposition 2** If the home government chooses the production subsidy rate optimally, then there does not exist a Nash equilibrium in which firms choose fragmentation even though the domestic production is socially desirable.

### 4 Concluding remarks

In this paper, we examine the welfare properties of strategic fragmentation under production subsidies. We consider a situation in which home firms strategically choose the location of production. There exists a Nash equilibrium in which home firms optimally choose to produce overseas even though domestic production is desirable in terms of the social welfare of the home country. If the home government provides a production subsidy for the domestic production by home firms, then it can reduce the possibility of socially undesirable fragmentation.

We first consider the case in which the subsidy rate for each output of domestic production is given exogenously. Under the fixed subsidy rate, socially undesirable fragmentation can still arise. Next, we examine the case of the optimal production subsidy. Given the locational choices by firms, the home government chooses the subsidy rate optimally. Then, there does not exist a Nash equilibrium in which home firms choose fragmentation but the
domestic production is socially desirable. Therefore, under the optimal production subsidy, socially undesirable fragmentation does not arise.

References


Figure 1: Nash equilibrium in the firm’s choice of a production location

Figure 2: Welfare properties of fragmentation under the fixed subsidy rate
Figure 3: Welfare properties of fragmentation under the optimal production subsidy rate