Vertical Trade and Free Trade Agreements*

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Abstract

We investigate the welfare effect and the incentive for free trade agreements (FTAs) in a vertical trade structure. We consider a three-country model in which an FTA is formed between a country exporting a final good whose production uses an intermediate good and a country exporting the intermediate good in exchange for the final good. Without external tariff reform by the FTA member, the FTA unambiguously decreases the nonmember country’s welfare, whereas it may or may not increase the FTA member countries’ welfare, depending on the initial tariff levels and the number of firms producing the intermediate and the final goods. In contrast, with external tariff reform, a Pareto-improving FTA is possible.

Keywords: Free trade agreements; Vertical trade; External tariff reform; Cournot competition

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1
1 Introduction

In the last two decades, free trade agreements (FTAs) have rapidly increased in the world economy. According to the Japanese Ministry of Economy, Trade and Industry (2006), there were 24 FTAs worldwide in 1986, but by 2006, there were 193. In particular, countries in Asia, the Middle East and Africa, which previously lacked FTAs, are now driving forward the formation of FTAs.

As the number of FTAs increases, studies on FTAs have become a hot issue in international trade theory. One research stream on FTAs clarifies their welfare effect (e.g., Kose and Riezman, 2000; Yi, 2000; Bond et al., 2004), and another stream addresses the incentives for FTAs (e.g., Kiyono, 1993; Raff, 2001; Nomura et al., 2006).

The previous literature focuses on horizontal trade, i.e., international trade in which countries mutually export final goods. However, vertical trade, i.e., international trade in which one country exports intermediate goods to other countries that process the imported inputs into final goods for their exports, is of growing importance in recent trade transactions. Such vertical trade relationships are of particular note in the Asian region. For example, Japan exports some parts such as semiconductors for computers to other Asian countries, including China, and imports computers mainly from China (JETRO, 2006). It is necessary to reevaluate FTAs, taking account of the vertical trade structure in the Asian region.

Thus, the purpose of this paper is to examine the welfare effect of FTAs in the case of vertical trade. To analyze this, we construct a simple three-
country model, where one country exports an intermediate good to the other two countries and imports a final good from them. We investigate how the formation of an FTA affects each country’s welfare. We have two scenarios in relation to the FTA; the first one is the case where external tariff reform between the FTA member countries and the nonmember country is not allowed under the conditions of the FTA; the second one is the case where external tariff reform is allowed. In the latter case, in particular, we consider a reduction of the FTA member country’s external tariff such that the trade volume between the FTA member country and the nonmember country is unchanged. In other words, we consider the Kemp-Wan-Ohyama-type tariff reduction (Ohyama, 1972; Kemp and Wan, 1976).

The assumption of a reduction of the external tariff by the FTA member country is reasonable from two viewpoints. First, from the institutional viewpoint, the GATT article XXIV prohibits the imposition of an external tariff that is more restrictive relative to the situation before the formation of the FTA. Following this article, we propose a reduction of the external tariff such that the trade volume is unchanged. Second, from the theoretical viewpoint, the tariff complementarity effect is well known in the studies on FTAs (e.g., Bagwell and Staiger, 1999). That is, the reduction of the external tariff makes the FTA member countries better off because it promotes competition between exporting countries. Therefore, it is quite reasonable to assume that the FTA member countries implement a reduction of the external tariff.

We have the following outcomes. Without external tariff reform between the FTA member and the nonmember countries, the FTA unambiguously makes the nonmember worse off, whereas it may or may not make the FTA
member countries better off, depending on the initial tariff levels and the number of firms producing the intermediate and the final goods. In contrast, if the FTA member can reduce the external tariff so that trade volume between the FTA member country and the nonmember country is unchanged, a Pareto-improving FTA is possible under certain conditions regarding the initial tariff levels and the number of firms producing the intermediate and the final goods.

As we noted above, the main focus of this paper is on the welfare effect of FTAs. Therefore, this paper aims at a contribution to the research on FTAs. Furthermore, this paper is related not only to the studies on FTAs but also to the studies on trade policy under vertical trade (e.g., Bernhofen, 1997; Ishikawa and Spencer, 1999; Chang and Sugeta, 2004; Yanase and Kawabata, 2008). Thus, our research links these two research streams, and provides a new insight into the studies on FTAs.

The rest of this paper is organized as follows. Section 2 presents the basic model and finds the equilibrium. Section 3 refers to the first scenario of the FTA. We focus on the incentives for an FTA without tariff reform between the FTA member and nonmember countries. Section 4 considers the second scenario, which allows us to reform the external tariff under the conditions of the FTA. Finally, section 5 concludes the paper.

2 The Model and the Preliminary Results

The world economy consists of three countries, A, B and C, between which a final good and an intermediate good are traded. There exist \( n \) identical firms producing the final good in country \( B \) and \( C \), respectively, whereas there are
no downstream firms in country $A$. The intermediate good, in contrast, is produced only in country $A$, which has $m$ identical firms. Moreover, we assume that consumption of the final good takes place only in country $A$. Therefore, the trade pattern is that country $A$ exports the intermediate good to countries $B$ and $C$, and imports the final good from these countries.

The government in country $A$ imposes a specific tariff, $T^A_i$, on imports of the final good from country $i$ ($i = B, C$), whereas the government in country $i$ imposes a specific tariff, $t_i$, on imports of the intermediate good from country $A$. Our setting is briefly illustrated in Figure 1.

The model involves two stages of decisions. In the first stage, upstream firms located in country $A$ compete in the Cournot way and sell the intermediate goods to downstream firms in countries $B$ and $C$. In the second stage, given the prices of the intermediate good, the downstream firms perform under Cournot competition in the final-good market in country $A$.\footnote{This setting implies that the downstream firms recognize their market power in the final-good market but act as price-takers in the intermediate-good market. This setting is often adopted in the literature on vertical trade (e.g., Bernhofen, 1997; Hwang, \textit{et al.}, 2007; Yanase and Kawabata, 2008). Ishikawa and Spencer (1999) provide a detailed discussion of the justification for this setting.} We derive the subgame-perfect equilibrium of this model in the following subsections.

<Insert Figure 1>

\section*{2.1 Equilibrium in the Final-Good Market}

We assume that producing one unit of the final good requires one unit of the intermediate good. Let $y_i$ and $Y$ be the output of a firm producing the final good (hereafter, we refer to the “final-good firm”) in country $i$ and the total output of the final good ($i = B, C$). Then, the profit of the final-good firm...
in country $i$ is given by

$$
\pi_i = [P(Y) - r_i - T_A^i] y_i,
$$

(1)

where $P(Y)$ is the inverse demand for the final good, and $r_i$ is the price of the intermediate good in the market in country $i$ ($i = B, C$). Firms choose output $y_i$ so as to maximize the profit (1), taking the outputs of rivals and the prices of the intermediate good as given. From equation (1), the first-order condition for profit maximization for the final-good firm in country $i$ is obtained as follows ($i = B, C$):

$$
P(Y) - r_i - T_A^i + P'(Y)y_i = 0.
$$

(2)

Note that $Y = \sum_{i=B,C} n\tilde{y}_i$. Condition (2) derives the Cournot-Nash equilibrium outputs in the final good market; $y_i = \tilde{y}_i(r_B, r_C, T_A^B, T_A^C)$ ($i = B, C$).

### 2.2 Equilibrium in the Intermediate-Good Market

Before analyzing the Cournot competition by the upstream firms in country $A$, we refer to the inverse demands for the intermediate good. Because the final-good firms are identical, the total demand for the intermediate input in country $i$ is $n\tilde{y}_i$ ($i = B, C$). Letting $X^i$ be the total export supply of the intermediate good to country $i$, the market-clearing condition in country $i$ is

$$
X^i = n\tilde{y}_i(r_B, r_C, T_A^B, T_A^C), \quad i = B, C.
$$

(3)

Equation (3) yields the inverse demand of the intermediate good, $r_i = \tilde{r}_i(X^B, X^C, T_A^B, T_A^C)$ ($i = B, C$).

We assume a constant marginal cost for producing the intermediate good, $k > 0$. Then, profit for a firm producing the intermediate good (hereafter,
the “intermediate-good firm”) in country A is expressed by

\[ \pi_A = \sum_{i=B,C} \left[ \tilde{r}_i(X^B, X^C, T^B_A, T^C_A) - k - t_i \right] x^i_A, \]  

(4)

where \( x^i_A \) is the intermediate-good firm’s supply to country \( i \) (\( i = B, C \)).

The firm determines \( x^B_A \) and \( x^C_A \) so as to maximize (4), taking the output of rival intermediate-good firms as given. From equation (4), the first-order conditions for profit maximization for the intermediate-good firm in country A are

\[ \tilde{r}_B(mx^B_A, mx^C_A, T^B_A, T^C_A) - k - t_B + \frac{\partial \tilde{r}_B}{\partial X^B} x^B_A + \frac{\partial \tilde{r}_C}{\partial X^C} x^C_A = 0, \]

\[ \tilde{r}_C(mx^B_A, mx^C_A, T^B_A, T^C_A) - k - t_C + \frac{\partial \tilde{r}_B}{\partial X^B} x^B_A + \frac{\partial \tilde{r}_C}{\partial X^C} x^C_A = 0. \]  

(5)

Conditions (5) determines the Cournot-Nash equilibrium outputs in the intermediate-good market as a function of the tariff rates; i.e., \( x^i_A = \tilde{x}^i_A(T^B_A, T^C_A, t_B, t_C) \) (\( i = B, C \)). Substituting this into the equilibrium outputs and prices in the second stage, we have the subgame-perfect equilibrium solutions as a function of the tariff rates. That is,

\[ r_i(T^B_A, T^C_A, t_B, t_C) \equiv \tilde{r}_i \left( mx^B_A(T^B_A, T^C_A, t_B, t_C), mx^C_A(T^B_A, T^C_A, t_B, t_C), T^B_A, T^C_A \right), \]

\[ y_i(T^B_A, T^C_A, t_B, t_C) \equiv \tilde{y}_i \left( r_B(T^B_A, T^C_A, t_B, t_C), r_C(T^B_A, T^C_A, t_B, t_C), T^B_A, T^C_A \right), \]

\[ p(T^B_A, T^C_A, t_B, t_C) \equiv P \left( Y(T^B_A, T^C_A, t_B, t_C) \right), \]

\( i = B, C \), where \( Y(T^B_A, T^C_A, t_B, t_C) \equiv \sum_{i=B,C} ny_i(T^B_A, T^C_A, t_B, t_C) \).

The welfare of country A consists of the sum of the profits of the intermediate-good firms, consumer surplus and country A’s tariff revenue. That is,

\[ W_A(T^B_A, T^C_A, t_B, t_C) = m\pi_A + CS + T^B_A ny_B + T^C_A ny_C, \]  

(6)
where \( CS = \int_0^Y p(s)ds - p(Y)Y \) is the consumer surplus. In contrast, the welfare function of country \( i \) consists of the sum of the profits of the final-good firms in country \( i \) and country \( i \)'s tariff revenue \((i = B, C)\). That is,

\[
W_i(T_B^A, T_C^A, t_B, t_C) = n\pi_i + t_i m x_A^i, \quad i = B, C. \tag{7}
\]

In the subsequent analysis, we assume a linear demand: \( p(Y) = \alpha - Y \), where \( \alpha > 0 \) is large enough to ensure positive outputs. Then, the uniqueness and stability of the Cournot-Nash equilibrium are guaranteed and the second-order conditions for profit maximization for any firms are satisfied in both the final-good and the intermediate-good markets. The subgame-perfect equilibrium solutions under the demand function appear in Table 1.\(^2\)

<Insert Table 1>

3 An FTA without External Tariff Reform

In the subsequent analysis, we consider an FTA between countries \( A \) and \( B \), and treat country \( C \) as a nonmember country.

In this section, we focus on the welfare effect of an FTA in the case where the FTA member countries and the nonmember country keep their tariff levels constant before and after the FTA. We discuss how the FTA affects the nonmember country’s welfare, and examine whether the FTA member

\(^2\)In Table 1, the equilibrium price of the intermediate good \( r_i \) is expressed as a function of \( T_A^i \) and \( t_i \) only \((i = B, C)\). The reason why \( r_i \) does not depend on \( T_A^j \) and \( t_j, j \neq i \), is that we assume a linear demand for the final good.
countries, A and B, have an incentive to form the FTA. We examine the welfare effects by comparing the pre-FTA and post-FTA welfare levels of these countries.

We assume that, before the formation of the FTA, country A levies a nondiscriminatory tariff on imports from countries B and C because of the principle of most-favored-nation treatment, i.e., \( T_A^B = T_A^C = T_0 > 0 \), and countries B and C impose the same level of tariff on imports from country A, i.e., \( t_B = t_C = t_0 > 0 \). After the FTA formation, it is assumed that countries A and B entirely eliminate the tariffs applying to trade between each other, i.e., \( T_A^B = t_B = 0 \), but that country A retains the tariff level on imports from country C, i.e., \( T_A^C = T_0 \). Moreover, country C does not change its tariff level on imports from country A, i.e., \( t_C = t_0 \).

Throughout the analysis in this section, we assume that the following condition is satisfied:

\[
t_0 < \frac{\alpha - k}{n + 1} - T_0. \tag{8}
\]

This condition guarantees that all intermediate-good and final-good firms produce positive outputs before and after formation of the FTA. That is, \( x_A^i > 0 \) and \( y_i > 0 \) are satisfied \( (i = B, C) \). Thus, this condition (8) often appears in the following as the positive-output condition.

The pre-FTA and post-FTA equilibria are shown in Table 2 and welfare levels in the pre-FTA and the post-FTA situations are shown in Tables 3 and 4, respectively. Thus, the welfare changes in each country as a result of the
formation of the FTA are obtained as follows:

\[
\Delta W_A \equiv W_A(0, T_0, 0, t_0) - W_A(T_0, T_0, t_0, t_0) = \frac{mn}{2(2n+1)^2(m+1)^2} f_A(T_0, t_0),
\]

(9)  

\[
\Delta W_B \equiv W_B(0, T_0, 0, t_0) - W_B(T_0, T_0, t_0, t_0) = \frac{mn}{(2n+1)^2(m+1)^2} f_B(T_0, t_0),
\]

(10)  

\[
\Delta W_C \equiv W_C(0, T_0, 0, t_0) - W_C(T_0, T_0, t_0, t_0) = -\frac{mn^2(T_0 + t_0)}{(2n+1)^2(m+1)^2} f_C(T_0, t_0),
\]

(11)  

where

\[
f_A(T_0, t_0) \equiv 2(\alpha - k) \left\{ (2n - m + 1)T_0 + 2(mn + 2n + 1)t_0 \right\} 
- (T_0 + t_0) \left\{ m(4n^2 + n - 2)T_0 + (3mn - 4n^2 + 2n + 2)t_0 \right\},
\]

\[
f_B(T_0, t_0) \equiv (\alpha - k) \left\{ 2m(n + 1)T_0 - (2n - m + 1)t_0 \right\} 
+ (T_0 + t_0) \left\{ m(n + 1)(n - 1)T_0 + (mn^2 + 2mn + 2n + 1)t_0 \right\},
\]

\[
f_C(T_0, t_0) \equiv 2m(\alpha - k) - m(n + 2)T_0 + (mn - m + 2n + 1)t_0.
\]

Focusing on the change in welfare in country C (equation (11)), we obtain the following result.\(^3\)

**Proposition 1**

*If any external tariff reforms are not allowed, the change in welfare of the nonmember country after the formation of the FTA (\(i.e., \Delta W_C\)) is unambiguously negative.*

\(^3\)All proofs of the upcoming Propositions appear in Appendices.
Thus, we find that the FTA between countries A and B is harmful for the nonmember country C in the absence of external tariff reforms.

We now turn to the welfare changes in the FTA member countries, A and B. Figures 2, 3 and 4 illustrate the changes in welfare in countries A and B from the pre-FTA to the post-FTA situation. The horizontal axis shows $T_0$, the initial tariff level imposed by country A, and the vertical axis describes $t_0$, the initial tariff level levied by countries B and C. Along the curve $\Delta W_A = 0$, country A’s welfare is unaffected by the FTA between countries A and B, i.e., $W_A(0, T_0, 0, t_0) = W_A(T_0, T_0, t_0, t_0)$. Above (resp. below) the curve $\Delta W_A = 0$, the FTA improves (resp. reduces) country A’s welfare. For similar reasons, along the curve $\Delta W_B = 0$, country B’s welfare is unaffected by the FTA, i.e., $W_B(0, T_0, 0, t_0) = W_B(T_0, T_0, t_0, t_0)$, whereas the FTA decreases (resp. increases) country B’s welfare on the left (resp. right) of the curve $\Delta W_B = 0$.

Figure 2 describes the case of $m = 1$, i.e., monopoly in the intermediate-good market. An FTA between countries A and B is always beneficial for country A. In contrast, it may or may not be beneficial for country B, depending on country A’s initial tariff level, $T_0$. If $T_0$ is low enough, the FTA is harmful to country B; otherwise, it is beneficial for country B.

<Insert Figure 2>

Figure 3 shows the case where $2 \leq m < 2n + 1$, i.e., there is a smaller number of intermediate-good firms relative to the number of final-good firms.

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4Here, instead of the mathematical explanations, we develop graphical explanations. The mathematical explanation appears in Appendix B.
In this case, the FTA may or may not be beneficial for both countries, depending on the initial tariff levels, $T_0$ and $t_0$. At least one of the countries will gain from the FTA in this case. There are some cases where the FTA is mutually beneficial for both countries, but no cases where neither of them becomes better off.

<Insert Figure 3>

Figure 4 illustrates the case where $2n + 1 \leq m$. i.e., there is a larger number of intermediate-good firms relative to the number of final-good firms. In this case, country $B$ always gains from the FTA. In contrast, it may or may not be beneficial for country $A$, depending on country $B$’s initial tariff level, $t_0$. If $t_0$ is low enough, the FTA is harmful to country $A$; otherwise, it is beneficial for country $A$.

<Insert Figure 4>

We summarize these results as Proposition 2.

**Proposition 2**

Suppose that tariff coordination between the FTA members and the non-member is not allowed. Suppose also that $T_A^B = T_A^C = T_0 > 0$ and $t_B = t_C = t_0 > 0$ hold under the pre-FTA situation, and that $T_A^B = t_B = 0$, $T_A^C = T_0$, and $t_C = t_0$ hold after an FTA is formed between countries $A$ and $B$. Then,

(i) For $m = 1$, if $T_0$ is sufficiently low, then country $B$ has no incentive to form the FTA. Otherwise, countries $A$ and $B$ have an incentive to form an FTA with each other.

(ii) For $2 \leq m < 2n + 1$, if $T_0$ is sufficiently high and $t_0$ is sufficiently low,
then country $A$ has no incentive to form the FTA. If $T_0$ is sufficiently low, country $B$ has no incentive to form the FTA. Otherwise, countries $A$ and $B$ have an incentive to form the FTA with each other.

(iii) For $2n + 1 \leq m$, if $t_0$ is sufficiently low, then country $A$ has no incentive to form the FTA. Otherwise, countries $A$ and $B$ have an incentive to form the FTA with each other.

The intuition behind Proposition 2 can be explained as follows.

First, the effect of the FTA on country $B$ is explained as follows: A decrease in $T_A^B$ directly reduces the marginal costs of the final-good firms in country $B$. In addition, a decrease in $t_B$ lowers the price of the intermediate good in country $B$ and reduces the marginal costs of the final-good firms in country $B$. Such reductions in the marginal costs increase the output of final-good firms in country $B$, thereby shifting some of the profits from the final-good firms in country $C$ to those in country $B$. These changes are summarized as the horizontal profit-shifting effect of the FTA.

On the other hand, a decrease in $T_A^B$ raises the demand for the intermediate good in country $B$, and thereby increases the price of the intermediate good, $r_B$. As payments to the intermediate-good firms increase, some of the profits are shifted from the final-good firms in country $B$ to the intermediate-good firms in country $A$. In addition, a decrease in $t_B$ worsens country $B$’s terms of trade on imports of the intermediate good, $r_B - t_B$. These changes are summarized as the vertical profit-shifting effect of the FTA.

As the number of intermediate-good firms in country $A$, $m$, becomes

\footnote{Country $B$’s import tariff, $t_B$, extracts the profits of the intermediate-good firms in country $A$. A decrease in $t_B$ implies that such profit extraction is weakened.}
smaller, the negative vertical profit-shifting effect becomes larger. In addi-
tion, as country A’s initial tariff, $T_0$, is lower, the positive horizontal profit-
shifting effect is weaker. Therefore, when the number of intermediate-good
firms is small and country A’s initial tariff is sufficiently low, country B loses
its incentive to form the FTA with country A.

Second, the effect of the FTA on country A is explained as follows. As
we saw above, decreases in $T_{BA}$ and $t_B$ increase the output of the final-good
firms in country B. This raises the demand for the intermediate good, and
thereby increases the outputs and profits of the intermediate-good firms in
country A. This is the vertical effect of the FTA. Furthermore, a decrease
in $t_B$ reduces the price of the final good, $p$, which induces an increase in
consumer surplus.

On the other hand, a decrease in $T_{BA}$ worsens country A’s terms of trade
on the final-good imports from country B, $p - T_{BA}$. In addition, decreases
in $T_{BA}$ and $t_B$ reduce the imports from country C, which reduces country A’s
tariff revenue.

As the number of intermediate-good firms increases, the positive vertical
effect becomes weaker. As the number of final-good firms decreases, the
negative terms of trade effect becomes larger. Further, as country B’s initial
tariff level, $t_0$, becomes lower, the positive effect from the elimination of $t_0$
becomes smaller. Therefore, when the number of intermediate-good firms
relative to the number of final-good firms is sufficiently large and country
B’s initial tariff level is sufficiently low, country A loses the incentive to form

\footnote{For similar reasons as stated in footnote 5, country A’s import tariff, $T_{BA}$, extracts the profits of the final-good firms in country B. A decrease in $T_{BA}$ implies that such profit extraction is weakened.}
the FTA with country $B$.

4 An FTA with External Tariff Reform

In the previous section, we found that the formation of an FTA between countries $A$ and $B$ makes the nonmember country $C$ unambiguously worse off, if tariff coordination between the FTA member and the nonmember countries is not allowed. In this section, we show the possibility of a Pareto-improving FTA in the present model by proposing an external tariff reform; i.e., a proportional reduction of the FTA member country’s external tariff level.

GATT Article XXVI 5(b) states that the levels of trade restrictions applied to parties not included in FTAs “shall not be higher or more restrictive than the corresponding duties and other regulations of commerce existing in the same constituent territories prior to the formation of the free-trade area”. We interpret this requirement as meaning that the trade volume between a member country of the FTA and a nonmember country is not reduced after the formation of the FTA. More specifically, we propose a reduction rate $\lambda$ such that country $C$’s export remains unchanged before and after the FTA.

Suppose that when signing the FTA with country $B$, country $A$ reduces its tariff on imports from country $C$ by $(1 - \lambda) \times 100\%$, where $\lambda \in (0, 1)$ is the rate of reduction. That is, country $A$ is assumed to set the post-FTA external tariff rate to $T^C_A = \lambda T_0$. The welfare changes in countries $A$, $B$ and
C are now given by

\[ \Delta W_A \equiv W_A(0, \lambda T_0, 0, t_0) - W_A(T_0, T_0, t_0), \]  
(12)

\[ \Delta W_B \equiv W_B(0, \lambda T_0, 0, t_0) - W_B(T_0, T_0, t_0), \]  
(13)

\[ \Delta W_C \equiv W_C(0, \lambda T_0, 0, t_0) - W_C(T_0, T_0, t_0). \]  
(14)

Let \( \bar{\lambda} \) be the rate of tariff reduction such that \( y_C(T_0, T_0, t_0, t_0) = y_C(0, \lambda T_0, 0, t_0). \)

Solving \( y_C(T_0, T_0, t_0, t_0) = y_C(0, \lambda T_0, 0, t_0) \) for \( \lambda \), we have

\[ \bar{\lambda} = \frac{T_0 - nt_0}{(n + 1)T_0}. \]  
(15)

Equation (14) is calculated as follows:

\[ \Delta W_C = \frac{mn\{(\lambda - 1)T_0 + nt_0 + \lambda T_0\}}{(m + 1)^2(2n + 1)^2} \times \left[-2m(\alpha - k) + m\{1 + (n + 1)\lambda\}T_0 + \{m - 1 - (m + 2)n\}t_0\right]. \]  
(16)

Substituting equation (15) into (16) and rearranging it, we have \( \Delta W_C = 0 \). That is, if the rate of external tariff reduction by country A is given by equation (15), the FTA between countries A and B accompanied by the reform of external tariffs not only does not reduce the volume of trade between country A (a FTA member) and country C (the nonmember), but also does not make the nonmember worse off.

Under \( \bar{\lambda} \), the positive-output condition changes from equation (8) to

\[ t_0 < \alpha - k - T_0. \]  
(17)

Because we consider the reduction of the external tariff by country A, the sign of \( \bar{\lambda} \) given by equation (15) must be nonnegative. This means that, in the following analysis, we focus on the set of initial tariffs \((T_0, t_0)\) such that
$T_0 - nt_0 \geq 0$ is satisfied. In the light of the positive-output condition (17), the set of $(T_0, t_0)$ must exist below the line $t_0 = \frac{1}{n}T_0$ in Figures 5 and 6. In addition, rewriting equation (15), we obtain

$$1 - \bar{\lambda} = \frac{n(T_0 + t_0)}{(n + 1)T_0} > 0.$$  \hspace{1cm} (18)

Equation (18) implies that we find a unique $\bar{\lambda}$ such that $0 \leq \bar{\lambda} < 1$.

Using the same method as in the previous section, let us discuss whether countries $A$ and $B$ have an incentive to sign the FTA if a proportional reduction in the external tariff $\bar{\lambda}$ is implemented.

We begin with country $B$. Substituting equation (15) into equation (13), the welfare change of country $B$ is calculated as follows:

$$\Delta W_B = \frac{mn}{(m + 1)^2(n + 1)^2(2n + 1)}g_B(T_0, t_0),$$  \hspace{1cm} (19)

where

$$g_B(T_0, t_0) = (n + 1)[2mT_0 - (mn - m + n + 1)t_0](\alpha - k) + (T_0 + t_0)[n(n + 2)(m + 1) + 1]t_0 - mT_0].$$

As in the previous section, we investigate the sign of $\Delta W_B$ by depicting a $W_B = 0$ curve in the $(T_0, t_0)$ space. As shown in Appendix C, the $\Delta W_B = 0$ curve is backward bending, and $\Delta W_B > 0$ (resp. $\Delta W_B < 0$) holds on the right (resp. left)-hand side of the curve.

It is clear from Figures 5 and 6 that $\Delta W_B > 0$ holds for any combination of $(T_0, t_0)$ satisfying $0 \leq \bar{\lambda} < 1$. That is, country $B$ always has an incentive to sign the FTA insofar as the reduction of the external tariff, i.e., $0 \leq \bar{\lambda} < 1$, is possible.
By substituting equation (15) into equation (12), Country A’s welfare change is analogously calculated as follows:

$$\Delta W_A = \frac{mn}{2(m + 1)^2(n + 1)^2(2n + 1)} g_A(T_0, t_0),$$

where

$$g_A(T_0, t_0) \equiv 2(n + 1) \{(2n - m + 1)T_0 + (mn + 3n + 2)t_0\}(\alpha - k)$$

$$+ (T_0 + t_0)\{m(2n^2 + 3n + 2)t_0 - (mn + 2n^2 + 4n + 2)T_0\}.$$ 

As for the case of country B, we can obtain a $\Delta W_A = 0$ curve in the $(T_0, t_0)$ space. As shown in Appendix C, the $\Delta W_A = 0$ curve is inverted U-shaped and $\Delta W_A > 0$ (resp. $\Delta W_A < 0$) holds above (resp. below) the curve.

From Figures 5 and 6, in the set of initial tariffs satisfying $0 \leq \bar{\lambda} < 1$, we see that country A unambiguously gains from signing the FTA, i.e., $\Delta W_A > 0$, if $m \leq 2n + 1$. If $m > 2n + 1$, the FTA may make country A worse off, i.e., $\Delta W_A < 0$, if the initial tariff levels $T_0$ and $t_0$ are sufficiently small; otherwise, the FTA makes country A better off.

Therefore, the analysis so far is summarized as Proposition 3.

**Proposition 3**

Consider the FTA between countries A and B with a proportional reduction in country A’s tariff level on imports from country C by $(1 - \bar{\lambda})$. If $m \leq 2n + 1$, countries A and B both have incentives to sign such an FTA. If $m > 2n + 1$, country B also has an incentive to sign the FTA, but country A may not be
willing to sign the FTA when the initial tariff levels $T_0$ and $t_0$ are sufficiently small.

Proposition 3 has two interpretations. Interpreting it positively, this proposition states that a Pareto-improving FTA is possible. At the same time, this proposition can be interpreted negatively; the Kemp-Wan-Ohyama theorem may or may not hold in the vertical trade structure depending on the initial tariff levels and the number of firms producing the intermediate and the final goods.

5 Concluding Remarks

We have investigated the welfare effect of an FTA and the incentives for signing an FTA in the case of vertical trade. We considered two scenarios: first, the case where external tariff reform is not allowed, and second, the case where external tariff reform is allowed. In particular, the second scenario focuses on an external tariff reduction by an FTA member, which maintains the nonmember country’s welfare at the same level before and after the FTA.

In the first scenario, we find that the FTA unambiguously makes the nonmember country worse off. In contrast, in the second scenario, we show that a Pareto-improving FTA is possible. The member country’s incentive for the FTA depends on the initial tariff levels and the number of intermediate-good and final-good firms.

Some extensions are available in our model. For instance, including firm entry or a determination of the trade pattern would enrich our analysis and provide additional interesting results. We would like to extend our model by
including these topics in our future research.

**Appendix A: Proof of Proposition 1**

From the positive-output condition (8), \( \alpha - k > (n + 1)(T_0 + t_0) \). Using this relation in \( f_C(T_0, t_0) \) of equation (11), we find that

\[
2m(\alpha - k) - m(n + 2)T_0 + (mn - m + 2n + 1)t_0 > 2m(n + 1)(T_0 + t_0) - m(n + 2)T_0 + (mn - m + 2n + 1)t_0 \\
= mnT_0 + (3mn + m + 2n + 1)t_0 > 0.
\]

Therefore, \( \Delta W_C < 0 \) holds.

**Appendix B: Proof of Proposition 2**

We begin with country \( B \). To obtain the \( \Delta W_B = 0 \) curve, we make equation (10) equal to zero.

The \( \Delta W_B = 0 \) curve intersects the \( t_0 \) axis twice. This is verified by solving \( f_B(0, t_0) = 0 \) for \( t_0 \). The intersections are derived as follows:

\[
(T_0, t_0) = (0, 0) \quad \text{and} \quad (T_0, t_0) = \left(0, \frac{(2n - m + 1)(\alpha - k)}{mn^2 + 2mn + 2n + 1}\right).
\]

Now, we compare the latter intersection with the intersection of the positive-output condition (equation (8)). Then,

\[
\frac{\alpha - k}{n + 1} < \frac{(2n - m + 1)(\alpha - k)}{mn^2 + 2mn + 2n + 1} \quad \text{if} \quad m = 1 \quad \text{and} \quad n \geq 3, \quad (A.1)
\]

\[
\frac{\alpha - k}{n + 1} > \frac{(2n - m + 1)(\alpha - k)}{mn^2 + 2mn + 2n + 1} \quad \text{otherwise}. \quad (A.2)
\]

Moreover, we have

\[
\frac{dt_0}{dT_0} \bigg|_{\Delta W_B=0; (T_0, t_0)=(0,0)} = -\frac{\partial f_B}{\partial T_0} = \frac{\partial f_B}{\partial t_0} = \frac{2m(n + 1)}{2n - m + 1}. \quad (A.3)
\]
From (A.1), (A.2) and (A.3), we find that
\[
\frac{dt_0}{dT_0} \bigg|_{\Delta W_B=0; (T_0, t_0)=(0,0)} > 0 \quad \text{if} \quad m < 2n + 1 \quad \text{(see Figures 2 and 3),}
\]
\[
\frac{dt_0}{dT_0} \bigg|_{\Delta W_B=0; (T_0, t_0)=(0,0)} < 0 \quad \text{if} \quad 2n + 1 < m \quad \text{(see Figure 4).}
\]

Note that \( \frac{dt_0}{dT_0} \bigg|_{\Delta W_B=0; (T_0, t_0)=(0,0)} = \infty \) if \( m = 2n + 1 \). In any case, the \( \Delta W_B = 0 \) curve is backward bending. Furthermore, on the right-hand (resp. left-hand) of the curve, \( \Delta W_B > 0 \) (resp. \( \Delta W_B < 0 \)) holds because
\[
\frac{\partial (\Delta W_B)}{\partial T_0} = \frac{mn}{(m+1)^2(2n+1)^2} \frac{\partial f_B}{\partial T_0} > 0.
\]

Next, we consider country \( A \). In order to obtain the \( \Delta W_A = 0 \) curve, we make equation (9) equal to zero. The \( \Delta W_A = 0 \) curve intersects the \( T_0 \) axis twice. This is verified by solving \( f_A(T_0, 0) = 0 \) for \( T_0 \). The intersections are derived as follows:
\[
(T_0, t_0) = (0, 0) \quad \text{and} \quad (T_0, t_0) = \left( \frac{2(2n - m + 1)(\alpha - k)}{m(4n^2 + n - 2)}, 0 \right).
\]

Now, we compare the latter intersection with the intersection of the positive-output condition (equation (8)). Then,
\[
\frac{\alpha - k}{n + 1} < \frac{(2n - m + 1)(\alpha - k)}{m(4n^2 + n - 2)} \quad \text{if} \quad m = 1, \quad \text{(A.4)}
\]
\[
\frac{\alpha - k}{n + 1} > \frac{(2n - m + 1)(\alpha - k)}{m(4n^2 + n - 2)} \quad \text{if} \quad m \geq 2. \quad \text{(A.5)}
\]

Moreover, we have
\[
\frac{dt_0}{dT_0} \bigg|_{\Delta W_A=0; (T_0, t_0)=(0,0)} = -\frac{\partial f_A/\partial T_0}{\partial f_A/\partial t_0} = -\frac{2n - m + 1}{2(mn + 2n + 1)}. \quad \text{(A.6)}
\]
From (A.4), (A.5), and (A.6), we find that

\[
\frac{dt_0}{dT_0} \bigg|_{\Delta W_A = 0; (T_0, t_0) = (0, 0)} < 0 \quad \text{if} \quad m < 2n + 1 \quad (\text{see Figures 2 and 3}),
\]

\[
\frac{dt_0}{dT_0} \bigg|_{\Delta W_A = 0; (T_0, t_0) = (0, 0)} \geq 0 \quad \text{if} \quad 2n + 1 \leq m \quad (\text{see Figure 4}).
\]

In any case, the \( \Delta W_A = 0 \) curve is U-shaped. Furthermore, above (resp. below) the curve, \( \Delta W_A > 0 \) (resp. \( \Delta W_A < 0 \)) holds because

\[
\frac{\partial (\Delta W_A)}{\partial t_0} = \frac{mn}{2(m + 1)^2(n + 1)^2(2n + 1)} \frac{\partial f_A}{\partial t_0} > 0.
\]

As a result of analyzing the shapes of the \( \Delta W_A = 0 \) and \( \Delta W_B = 0 \) curves, we have the following results:

(i) For \( m = 1 \), in the case of a sufficiently low \( T_0 \), country \( B \) has no incentive to form the FTA;

(ii) For \( 2 \leq m < 2n + 1 \), in the case of a sufficiently high \( T_0 \) and a sufficiently low \( t_0 \), country \( A \) has no incentive to form the FTA. In the case of a sufficiently low \( T_0 \), country \( B \) has no incentive to form the FTA; and.

(iii) For \( 2n + 1 \leq m \), in the case of a sufficiently low \( t_0 \), country \( A \) has no incentive to form the FTA with country \( B \).

In the case of the other sets of initial tariffs \( (T_0, t_0) \), both countries \( A \) and \( B \) have an incentive to form the FTA with each other. ■

**Appendix C: Proof of Proposition 3**

In this part, we follow the same reasoning as in Appendix B. We begin with country \( B \). To obtain the \( \Delta W_B = 0 \) curve, we make equation (13) equal to zero.
The $\Delta W_B = 0$ curve intersects the $t_0$ axis twice. This is verified by solving $g_B(0, t_0) = 0$ for $t_0$. The intersections are derived as follows:

$$(T_0, t_0) = (0, 0) \text{ and } (T_0, t_0) = \left( 0, \frac{(mn - m + n + 1)(n + 1)(\alpha - k)}{n(m + 1)(n + 2) + 1} \right).$$

We compare the latter intersection with the intersections of the positive-output condition (equation (17)) and find that the intersection of the positive-output condition $\alpha - k$ is greater than the latter intersection.

Moreover, we have

$$\left. \frac{dt_0}{dT_0} \right|_{\Delta W_B = 0; (T_0, t_0) = (0, 0)} = -\frac{\partial g_B / \partial T_0}{\partial g_B / \partial t_0} = \frac{2m}{mn - m + n + 1}. \quad (A.7)$$

From (A.7), the $\Delta W_B = 0$ curve is upward sloping at the origin, and is backward bending. Furthermore, on the right-hand (resp. left-hand) of the curve, $\Delta W_B > 0$ (resp. $\Delta W_B < 0$) holds because

$$\frac{\partial (\Delta W_B)}{\partial T_0} = \frac{mn}{(m + 1)^2(n + 1)^2(2n + 2)} \frac{\partial g_B}{\partial T_0} > 0.$$

It is clear from Figures 5 and 6 that $\Delta W_B > 0$ for any pair $(T_0, t_0)$ such that $0 \leq \bar{\lambda} < 1$ holds. That is, country $B$ always has an incentive to sign the FTA if the initial tariffs satisfy $0 \leq \bar{\lambda} < 1$.

Next, we consider country $A$. To obtain the $\Delta W_A = 0$ curve, we make equation (12) equal to zero.

The $\Delta W_A = 0$ curve intersects the $T_0$ axis twice. This is verified by solving $g_A(T_0, 0) = 0$ for $T_0$. The intersections are derived as follows:

$$(T_0, t_0) = (0, 0) \text{ and } (T_0, t_0) = \left( \frac{2(m - 2n + 1)(n + 1)(\alpha - k)}{m(2n^2 + 3n + 2)}, 0 \right).$$
Now, we compare the latter intersection with the intersection of the positive-output condition (equation (17)). Then,

$$\alpha - k > \frac{2(m - 2n + 1)(n + 1)(\alpha - k)}{m(2n^2 + 3n + 2)}.$$  \hfill (A.8)

Moreover, we have

$$\frac{dt_0}{dT_0} \bigg|_{\Delta W_A = 0; (T_0, t_0) = (0, 0)} = -\frac{\partial g_A}{\partial T_0} = -\frac{2n - m + 1}{mn + 3n + 2},$$  \hfill (A.9)

From (A.8) and (A.9), we find that

$$\frac{dt_0}{dT_0} \bigg|_{\Delta W_A = 0; (T_0, t_0) = (0, 0)} \leq 0 \quad \text{if} \quad m \leq 2n + 1 \quad \text{(see Figure 5)},$$

$$\frac{dt_0}{dT_0} \bigg|_{\Delta W_A = 0; (T_0, t_0) = (0, 0)} > 0 \quad \text{if} \quad 2n + 1 < m \quad \text{(see Figure 6)}.$$  

In any case, the $\Delta W_A = 0$ curve is inverted U-shaped as illustrated by Figures 5 and 6. Furthermore, above (resp. below) of the curve, $\Delta W_A > 0$ (resp. $\Delta W_A < 0$) holds because

$$\frac{\partial (\Delta W_A)}{\partial t_0} = \frac{mn}{2(m + 1)^2(n + 1)^2(2n + 1)} \frac{\partial g_A}{\partial t_0} > 0.$$  

As a result of analyzing the shapes of the $\Delta W_A = 0$ and $\Delta W_B = 0$ curves, we have the following results:

(i) For $m \leq 2n + 1$, countries $A$ and $B$ both have incentives to form the FTA with each other; and

(ii) For $2n + 1 < m$, country $A$ may have no incentive to form the FTA.  

24
References


Table 1: Subgame perfect equilibrium

\[
\begin{align*}
  x^B_A(T^B_A, T^C_A, t_B, t_C) &= \frac{n}{(m + 1)(2n + 1)} \left\{ \alpha - k - (n + 1)T^B_A + nT^C_A - (n + 1)t_B + nt_B \right\} \\
  x^C_A(T^B_A, T^C_A, t_B, t_C) &= \frac{n}{(m + 1)(2n + 1)} \left\{ \alpha - k + nT^B_A - (n + 1)T^C_A + nt_B - (n + 1)t_C \right\} \\
  r_B(T^B_A, t_B) &= \frac{1}{(m + 1)} \left( \alpha + mk - T^B_A + mt_B \right) \\
  r_C(T^C_A, t_C) &= \frac{1}{(m + 1)} \left( \alpha + mk - T^C_A + mt_C \right) \\
  y^B_A(T^B_A, T^C_A, t_B, t_C) &= \frac{m}{(m + 1)(2n + 1)} \left\{ \alpha - k - (n + 1)T^B_A + nT^C_A - (n + 1)t_B + nt_C \right\} \\
  y^C_A(T^B_A, T^C_A, t_B, t_C) &= \frac{m}{(m + 1)(2n + 1)} \left\{ \alpha - k + nT^B_A - (n + 1)T^C_A + nt_B - (n + 1)t_C \right\} \\
  p(T^B_A, T^C_A, t_B, t_C) &= \frac{1}{(m + 1)(2n + 1)} \left\{ (m + 2n + 1)\alpha + 2mnk + mnT^B_A + mnT^C_A + mnt_B + mnt_C \right\}
\end{align*}
\]
### Table 2: The pre-FTA and post-FTA equilibrium

<table>
<thead>
<tr>
<th></th>
<th>pre-FTA</th>
<th>post-FTA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_A^B$</td>
<td>$\frac{n(\alpha - k - T_0 - t_0)}{(m + 1)(2n + 1)}$</td>
<td>$\frac{n(\alpha - k + nT_0 + nt_0)}{(m + 1)(2n + 1)}$</td>
</tr>
<tr>
<td>$x_A^C$</td>
<td>$\frac{n(\alpha - k - T_0 - t_0)}{(m + 1)(2n + 1)}$</td>
<td>$\frac{n(\alpha - k - (n + 1)T_0 - (n + 1)t_0)}{(m + 1)(2n + 1)}$</td>
</tr>
<tr>
<td>$y_B$</td>
<td>$\frac{m(\alpha - k - T_0 - t_0)}{(m + 1)(2n + 1)}$</td>
<td>$\frac{m(\alpha - k + nT_0 + nt_0)}{(m + 1)(2n + 1)}$</td>
</tr>
<tr>
<td>$y_C$</td>
<td>$\frac{m(\alpha - k - T_0 - t_0)}{(m + 1)(2n + 1)}$</td>
<td>$\frac{m(\alpha - k - (n + 1)T_0 - (n + 1)t_0)}{(m + 1)(2n + 1)}$</td>
</tr>
<tr>
<td>$r_B$</td>
<td>$\frac{\alpha + mk - T_0 + mt_0}{m + 1}$</td>
<td>$\frac{\alpha + mk}{m + 1}$</td>
</tr>
<tr>
<td>$r_C$</td>
<td>$\frac{\alpha + mk - T_0 + mt_0}{m + 1}$</td>
<td>$\frac{\alpha + mk - T_0 + mt_0}{m + 1}$</td>
</tr>
<tr>
<td>$p$</td>
<td>$\frac{(m + 2n + 1)\alpha + 2mnk + 2mnT_0 + 2mnt_0}{(m + 1)(2n + 1)}$</td>
<td>$\frac{(m + 2n + 1)\alpha + 2mnk + mnT_0 + mnt_0}{(m + 1)(2n + 1)}$</td>
</tr>
</tbody>
</table>
Table 3: Welfare levels in the pre-FTA situation

\[
W_A(T_0, T_0, t_0, t_0) = \frac{2mn}{(m+1)(2n+1)} T_0 (\alpha - k - T_0 - t_0) \\
+ \frac{2mn}{(m+1)^2(2n+1)} (\alpha - k - T_0 - t_0)^2 + \frac{2m^2n^2}{(m+1)^2(2n+1)^2} (\alpha - k - T_0 - t_0)^2
\]

\[
W_B(T_0, T_0, t_0, t_0) = \frac{mn}{(2n+1)(m+1)} t_0 (\alpha - k - T_0 - t_0) + \frac{m^2n}{(m+1)^2(2n+1)^2} (\alpha - k - T_0 - t_0)^2
\]

\[
W_C(T_0, T_0, t_0, t_0) = \frac{mn}{(m+1)(2n+1)} t_0 (\alpha - k - T_0 - t_0) + \frac{m^2n}{(m+1)^2(2n+1)^2} (\alpha - k - T_0 - t_0)^2
\]

Table 4: Welfare levels in the post-FTA situation

\[
W_A(0, T_0, 0, t_0) = \frac{mn}{(m+1)(2n+1)} T_0 (\alpha - k - (n+1)T_0 - (n+1)t_0) \\
+ \frac{m^2n^2}{2(m+1)^2(2n+1)^2} (2\alpha - 2k - T_0 - t_0)^2 + \frac{mn(n+1)}{(m+1)^2(2n+1)^2} (\alpha - k + nT_0 + nt_0)^2 \\
+ \frac{mn(n+1)}{(m+1)^2(2n+1)^2} (\alpha - k - (n+1)T_0 - (n+1)t_0)^2 \\
+ \frac{m^2n^2}{(m+1)^2(2n+1)^2} (\alpha - k + nT_0 + nt_0)(\alpha - k - (n+1)T_0 - (n+1)t_0)
\]

\[
W_B(0, T_0, 0, t_0) = \frac{m^2n}{(m+1)^2(2n+1)^2} (\alpha - k + nT_0 + nt_0)^2
\]

\[
W_C(0, T_0, 0, t_0) = \frac{mn}{(m+1)(2n+1)} t_0 (\alpha - k - (n+1)T_0 - (n+1)t_0) \\
+ \frac{m^2n}{(m+1)^2(2n+1)^2} (\alpha - k - (n+1)T_0 - (n+1)t_0)^2
\]
Figure 1: Market Structure
Figure 2: the case without the external tariff reform for \( m = 1 \)
Figure 3: the case without the external tariff reform for $2 \leq m < 2n + 1$
Figure 4: the case without the external tariff reform for $2n+1 < m$
Figure 5: the case with the external tariff reform for \( m \leq 2n + 1 \)
$6_0 = \Delta_{BW k}$

Figure 6: the case with the external tariff reform for $2n + 1 < m$