学位申請論文要旨

題目: "ESSAYS ON SEASONALLY AND FRACTIONALLY DIFFERENCED TIME SERIES"

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Over the past decade, there has been much interest in many time series such as hydrogical and financial time series with the property of strong dependence. These series have the property of slowly declining serial correlations and the sum of their serial correlations may diverge. Typically, a scalar time series, $\{y_t\}$, whose autocovariances $\gamma(j)$ can be approximated by

$$\gamma(j) = O(\kappa(j) j^{2d-1}) \qquad \text{as } j \to \infty.$$

where $d < 0.5$ and $|\kappa(j)| < C$ for some positive constant $C > 0$. This means that its autocovariances decay hyperbolically and that the process has a long memory ($\sum_j |\gamma(j)| = \infty$) or an intermediate memory ($\sum_j |\gamma(j)| < \infty$). While stationary autoregressive moving average (ARMA) models do not display this behavior, their autocorrelations decay exponentially, *i.e.*, $\gamma(j) = O(\alpha^j)$, where $\alpha \in (0,1)$. Useful surveys of examples of long memory processes, and of models and methods for dealing with data that has long-range dependence, can be found in Beran (1994).

Various new classes of time series that have the property of strong dependence are discussed by Granger and Joyeux (1980), Hosking (1981), and Gray *et al*. (1988), which extend the seasonally integrated autoregressive moving average (SARIMA) model of Box and Jenkins (1976). Extensions allowed the differencing parameter to take on fractional values. The models introduced by Granger and Joyeux (1980) and Hosking (1981), known as fractionally integrated autoregressive moving average (ARFIMA) models, which extend the integrated autoregressive moving average (ARIMA) models of Box and Jenkins (1976), allow the spectral density to be unbounded and peak at zero frequency. The models introduced by Gray *et al*. (1988), known as Gegenbauer and Gegenbauer autoregressive moving average (GARMA) models, allow the spectral density to be unbounded and peak at an arbitrary frequency $\omega \in [0, \pi]$. Recently, Woodward *et al*. (1998) generalized GARMA models, known as k-factor GARMA models, to allow the spectral density to be unbounded and peak at an arbitrary k different frequencies $\omega \in [0, \pi]$.

The purpose of this thesis is to further develop the properties of the long memory models studied by Woodward *et al*. (1998) and to discuss the asymptotic prediction mean squared error of a linear predictor for the long memory processes.

In Chapter 1 we deal with k-factor GARMA models. After investigating the properties of k-factor GARMA models, we discuss estimation and the testing problem for these models. We note that since the class of time series models of k-factor GARMA models contains the class of flexible models by Hassler (1994), our results can be applied to these flexible models. Chapter 2 investigates one more special case of the k-factor GARMA model, which is considered by Porter-Hudak (1990) and naturally extends the SARIMA models by allowing the two differencing parameters to take on fractional values. We call this model a seasonally and fractionally integrated ARMA (SARFIMA) model. Similarly to Chapter 1, we mainly discuss estimation of, and the testing problem in, SARFIMA models. In Chapter 1 and Chapter 2 we examined the asymptotic properties of the estimators and statistics without assuming normal and stationary time series. It was shown that the standard asymptotic results hold for the tests and the estimators; that is, CSS estimators are strongly consistent and tend towards normality with $O_p(1/\sqrt{T})$, LM test statistics are more powerful than the old Portmanteau test statistics, and Godfrey's LM test is also applicable. The finite behavior of the tests and estimators was also examined by simulations, and the source of differences in behavior was made clear in terms of the asymptotic theory.

Chapter 3 dealt with prediction theory of nonstationary long memory processes, referred to as the ARFISMA model by Hassler (1994). After investigating the general theory relating to convergence of the moments of the nonlinear least squares estimators, we evaluated the asymptotic prediction mean squared error of two predictors. One is defined by the estimator of the differencing parameter and the other is defined by a fixed differencing parameter, which is, in other words, a parametric predictor of the SARIMA models. The effects of misspecifying the integration order in the ARFISMA model were clarified by the asymptotic results relating to the prediction mean squared error. The finite sample behavior of the predictor was examined by simulations, and the source of differences in behavior was made clear in terms of the asymptotic theory. All proofs are given in the Appendix.

References

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