**Summary of Doctoral Dissertation:**
**"Voluntary Participation Games in Public Good Mechanisms: Coalitional Deviations and Efficiency"**

**Ryusuke Shinohara**

## 1. Motivation

The purpose of this dissertation is to examine a participation problem in a mechanism to produce a (pure) public good. The public good is one which satisfies non-excludability and non-rivalry: all agents can consume the same amount of a public good regardless of their contribution to it. Therefore, every agent has an incentive to free-ride the public good that is produced by other agents. As a result, the public good is provided at a low level. This problem is known as the "free-rider" problem.

A solution to the free-rider problem is the construction of economic mechanisms or systems in which the socially efficient level of public goods is provided as a result of strategic behavior of agents. The construction of such mechanisms has been studied in two distinct directions: one is *strategy-proofness*, and the other is *Nash implementation*. In the theory of strategy-poofness, the mechanism designer, for example, the policy-implementation organization or the supplier of public goods, constructs a mechanism to elicit information about agents' preferences; this information is necessary for the provision of an efficient level of public goods as well as efficient cost distribution. However, preferences are usually privately known. Therefore, it is possible that selfish agents will try to misrepresent their preferences in order to manipulate the provision of the public good and their cost burdens. As a result of such strategic misrepresentation, the level of the public good may be socially inefficient. Thus, the construction of procedures that give all agents an incentive to announce their true preferences is important in the provision of public goods. There have been many studies on mechanisms in which the truthful revelation of preferences is a weakly dominant strategy for all agents; such mechanisms are referred to as *strategy-proof* mechanisms. In the strategy-proof mechanisms, some amount of private goods must be discarded to give incentives to agents to reveal their true preferences. Thus, strategy-proofness and Pareto efficiency are incompatible.

Nash implementation is the approach that employs Nash equilibria as equilibrium concepts. In this theory, mechanisms are constructed in such a way as to achieve Pareto efficient allocations or Lindahl allocations. Many authors such as Groves d Ledyard (1977) and Walker (1981) have constructed such mechanisms and succeeded in supporting the efficient allocations as Nash equilibria of the mechanisms. Therefore, the construction of mechanisms can solve the free-rider problem in this theory.

However, implementation theory has limited the discussion to the construction of the public goods mechanism, and the participation problem in the mechanism has not been sufficiently studied. In the implementation theory, all agents are assumed to participate in the mechanism. Hence, the results in this field indicate that the public good can be provided efficiently by constructing the public good mechanism under the assumption of the participation of all agents. Although the assumption is essential for the mechanism to fulfill its function in the economy, few studies have examined whether or not agents enter the mechanism voluntarily. The participation problem is also important from practical points of view. As we know from the real world examples such as the Kyoto Protocol and NHK, there are many situations in which the participation problem has serious effects on the effectiveness of public good mechanisms.

Palfrey and Rosenthal (1984) and Saijo and Yamato (1999) pointed out the importance of the strategic behavior of agents as they decide whether or not to participate in the mechanisms. Palfrey and Rosenthal (1984) formulated a participation game with a public project. In this game, all agents simultaneously choose either participation or non-participation, and only the agents that choose participation bear the cost of the public project. Thus, non-participants can benefit from the project at no cost. Saijo and Yamato (1999) considered participation games with a perfectly divisible public good. These authors showed that not all agents enter the mechanism in Nash equilibria of the game. These results indicate that the free-ride problem with respect to the participation decision occurs and severely affects resource allocations of the economy.

The existing literature has not considered the possibility that agents form a coalition and coordinate the participation decisions. Researchers have characterized a set of participants that is stable against unilateral deviations of agents, focusing solely on subgame perfect Nash equilibria or Nash equilibria. However, in the theory of implementation, the mechanisms have been constructed not only under the concept of Nash equilibrium but also under other equilibrium concepts such as coalition-proof equilibria (Bernheim, Peleg, and Whinston, 1987) and strong equilibria (Aumann, 1959). If a mechanism is constructed under the assumption that agents form coalitions, then it is natural to consider that agents also coordinate participation decisions. Hence, in this case, it is important to analyze the participation decision, considering the possibility of the cooperative behavior of agents. In addition, behavior is based on various behavioral principles, and economists do not know which behavioral principles people will actually employ. Thus, considering the various possibilities is meaningful for understanding the consequences of strategic behavior.

In this dissertation, we consider the possibility that agents form a coalition in the participation decision stage. We analyze the participation problem in a public good mechanism, as in Saijo and Yamato (1999). We examine coalition-proof equilibria and strong equilibria of the participation decision stage game. The main purpose of this dissertation is to clarify whether there are coalition-proof equilibria and strong equilibria in this game and to characterize these two equilibria of this game, if such two equilibria exist.

## 2. Participation Games with a Perfectly Divisible Public Good

In Part I, we consider the participation game in a mechanism to produce a perfectly divisible public good. In Chapter 2, we consider the case in which agents' preferences are identical and examine the coalition-proof equilibria of this game. We provide an example in which the participation game has multiple Nash equilibria that support the different numbers of participants. However, we show that coalition-proof equilibria exist and only the maximal number of participants in the set of Nash equilibria is supportable as a coalition-proof equilibrium. We further show that the set of coalition-proof equilibria coincides with the Pareto efficient frontier of the set of Nash equilibria. Since the definition of coalition-proof equilibria is very complicated, little is known about properties of this equilibrium. The contribution of this chapter is to show the existence of coalition-proof equilibria and to clarify the properties of this equilibrium in the participation game under the assumption of identical agents.

In Chapter 3, we extend the analysis in Chapter 2 to the case of heterogeneous agents. In this chapter, we investigate the number of participants that is attained at coalition-proof equilibria. In the case of heterogeneous agents, the number of participants in coalition-proof equilibria may be multiple, differently from the case of identical agents. The main result of this chapter is to provide a sufficient condition under which the number of participants in coalition-proof equilibria is unique. As a result, we confirm that the unique number of participants can be achieved not only in the case of identical agents but also in some cases of heterogeneous agents.

In Chapter 4, we study coalition-proof equilibria based on two different dominance relations: strict dominations and weak dominations. A coalition deviates only if all members of the coalition can be better off by switching their strategies under the notion of strict domination, while a coalition deviates if all members of the coalition are not worse off and at least one of the members is better off by changing their strategies under the notion of weak domination. In equilibrium concepts based on coalition deviations such as the core and strong equilibria, the set of equilibria under weak domination is included in that under strict domination. However, the set of coalition-proof equilibria under strict domination and that under weak domination are not necessarily related by inclusion. We show that if a game satisfies the conditions of anonymity, monotone externality, and strategic substitutability, then the set of coalition-proof equilibria under weak domination is included that under strict domination. Since this class of games contains many interesting games such as the participation games studied in Chapter 2 and the Cournot oligopoly game, the inclusion relation holds in many games studied in economics.

## 3. Participation Games with a Discrete Public Good

In Part II, we consider the participation game in a mechanism to produce a discrete public good. In Chapter 5, we consider the participation game in a mechanism to implement a public project. The mechanism implements the allocation rule that satisfies Pareto efficiency, individual rationality, and the condition that every participant bears positive cost shares. In this game, both efficient and inefficient allocations are supportable as Nash equilibria. We examine strict Nash equilibria, coalition-proof equilibria, and strong equilibria of this game. We show that, in the participation game with a public project, (1) the set of strict Nash equilibria, that of strong equilibria, and that of coalition-proof equilibria coincide, (2) the three equilibria exist, and (3) only the efficient allocations are supportable as the three equilibria. These results are in contrast with those of Saijo and Yamato (1999). Saijo and Yamato (1999) showed that there are no Nash equilibria to support the efficient allocations when the public good is perfectly divisible in many cases. However, we prove that the efficient allocation is achieved in a Nash equilibrium if the level of the public good is of a fixed size. This is a contribution of this chapter. Also, these results are new findings in the following respects: The existence of strong equilibria and the equivalence between strong equilibria and coalition-proof equilibria have been studied in the theory of the provision of local public goods. In the case of non-excludable public goods, the existence and the equivalence have not been examined. One of the contributions of this dissertation is to provide sufficient conditions for the equivalence and the existence in the case of non-excludable public goods.

In Chapter 6, we study participation games with a multiple-choice public good; the public good is produced in integer units and at most two unit of the public good are provided. We consider the participation game in a mechanism to implement the proportional cost-sharing rule. In this case, unlike in the case of a public project, there are not necessarily Nash equilibria that attain Pareto efficiency. We show that no Nash equilibria support efficient allocations if agents are identical and a mild condition is satisfied. These findings indicate that the results in the case of a multiple-choice public good differ from those in the case of a public project, and that the inefficiency stemming from the agents' strategic behavior with respect to the participation depends on the number of alternatives that consist of the public good space.

## 4. Conclusion

Table 1 on page 8 is a summary of the main results of this dissertation regarding the existence of equilibria. In this table, each column represents the public good space and each row represents the notion of equilibria. We confirm from Table 1 that Nash equilibria supporting efficient allocations and strong equilibria do not exist in the case of a perfectly divisible public good and that of a multiple choice public good. The results in the two cases indicate a similar tendency. On the other hand, both equilibria exist in the case of a public project. In conclusion, we can say that the inefficiency of Nash equilibria and the non-existence of strong equilibria are due to the setting that the level of public good can take multiple positive values.

In the cases of a perfectly divisible public good and a discrete and multi-unit public good, the coalition-proof equilibria exist, although there is no strong equilibrium. From the results of Chapter 2, the set of coalition-proof equilibria coincides with the Pareto-efficient frontier of the set of Nash equilibria. This indicates that agents have an incentive to coordinate their participation decisions at an inefficient Nash equilibrium when agents can form a coalition. As a result, the coordination leads to the Pareto-efficient frontier of the set of Nash equilibria and the allocative efficiency improves. The same applies to the cases of a perfectly divisible public good and a multiple-choice public good. Note that the improvement of payoffs to members of coalitions does not necessarily imply the improvement of efficiency in allocations. Thus, the possibility of coalitional deviations improves the efficiency of equilibrium allocations in the class of participation games.

Chapter 7 concludes this dissertation and provides future prospects.

| | $\mathbb{R}_+$ (identical agents) | $\{0,1\}$ | $\{0,1,2\}$ (identical agents) |
|---|---|---|---|
| Nash equilibrium achieving the efficiency | − | + | − |
| Coalition-proof equilibrium | + | + | + |
| Strong equilibrium | − | + | − |

Table 1: Existence of equilibria. Symbol + means the existence of the equilibrium and − indicates non-existence of the equilibrium.

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