ABSTRACT of DISSERTATION

Submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Economics,

Graduate School of Economics, Hitotsubashi University.

On the Recovery Process Models

by

Yuki Itoh

November, 2009

Chair: Hajime Takahashi

Co-Chairs: Yoichi Kuwana, Junichiro Fukuchi, Eiji Kurozumi, and Tsunemasa Shiba

With the arrival of a long recession called the "lost decade" in Japan following the collapse of the Japanese asset price bubble and more recently the subprime mortgage crisis, the number of defaults by small companies is increasing. The importance of the quantification of the credit risk has grown. Moreover, because of the implementation of Basel Ⅱ, it is very important for financial institutions to establish quantification methods for determinating the credit risk of the debts of companies. In other words, if banks calculate their own regulatory capital requirements by using the Advanced Measurement Approaches of Basel Ⅱ, they should estimate three parameters, namely, Probability of Default (PD), Loss Given Default (LGD $= 1-$recovery rate), and Exposure At Default (EAD) for each company.

In this situation, many studies have been conducted on the quantification of the credit risk. However, although many studies have been conducted on the quantification of the credit risk, few studies have been

conducted to analyze the recovery of debt. In this dissertation, we study the quantification of the recovery rate for the debt of individual companies. Moreover, we extend a one-company model to a two-companies model. We focus only on the recovery rate for the debt (mainly bank loans) of small companies, which cannot be traded or is traded with strong constraint in the market. Therefore, the market recovery rate, which is the market price of the debt at default divided by the face value of the debt is unknown and thus cannot be used for the quantification of the recovery rate. In the first place, there is no defaulting debt market in Japan.

In particular, few studies have been conducted on the recovery of the debt (on bank loan) of an individual company, because most of the recovery data are kept confidential by lenders and borrowers. Thus, empirical data is seldom available for academic studies. To the best of our knowledge, Itoh and Yamashita (2008) is the only paper that studies empirically how the recovery progresses (the relationship between time and recoveries) for each defaulting company. In Itoh and Yamashita (2008), both the recovery points and the increments of recoveries are analyzed. It follows from Itoh and Yamashita (2008) that we may assume that (1) there are multiple recovery points for the same debt, (2) the recovery points are random, and (3) the increments of recoveries are random (see Figures 6, 7, and 8 in Itoh and Yamashita (2008)). Based on these assumptions, we refer to the process of the recovery points and the increments of recoveries as the recovery process. As a first step, we model the recovery process for a single company by a compound Poisson process, the most basic marked point process, which has these three properties.

Single Company Model In Chapter 3, on the basis of the study of Itoh and Yamashita (2008), we first model the recovery process for the debt of a single defaulting company by a homogeneous compound Poisson process.

In general, the debt continues to bear interest after default, and then the real values of early and late recovery are different, even if nominal values of early and late recovery are the same. Moreover, Itoh and Yamashita (2008) show that there are multiple recovery points for the same debt. In the light of these facts, we will suppose that the debt bears interest after default and that there are multiple recovery points for the same debt. These three properties distinguish our models from the preexisting ones. Existent recovery models do not consider more than one recovery point. Thus, they cannot distinguish the increment of recovery (each recovery) from the cumulative recovery, whereas, our model can analyze the relationship between the cumulative recovery, the increment of recovery, the initial debt amount (debt amount at default), the prescription (the time required for last possible recovery), and the interest rate. Moreover, our model is able to estimate LGD in Basel Ⅱ and we develop a method for the same. It is one of the main purposes of this dissertation.

Itoh and Yamashita (2008) point out that the rate of change in the empirical cumulative recovery rate is diminishing in time. In the homogeneous compound Poisson model, the rate of change in the cumulative recovery rate is constant in time. Therefore, we extend our model to an inhomogeneous compound Poisson process. If we model the intensity as a decreasing function of time, the rate of change in the cumulative recovery becomes a diminishing function of time.

We derive the expected value and the variance of the survival value of debt and recovery rate, and also derive the probability distribution function and the expected value of the recovery completion time. Moreover, in the homogeneous compound Poisson model, we present the numerical methods for calculating transition density, the expected value, and the variance based on the Panjer recursion formula and the fast Fourier transformation. We conduct numerical experiments on the above characteristics. Further, we compare the numerical accuracy and the speed of the Panjer recursion formula and the fast Fourier transformation. The accuracy is the same, but the speed of numerical calculation of the Panjer recursion formula is faster than that of the fast Fourier transformation.

Because there has been no study on the numerical method to calculate the transition density of an inhomogeneous compound Poisson process, we develop a new method of calculating it. Our method is based on approximating an inhomogeneous compound Poisson process by a piecewise homogeneous compound

Poisson process. This method is applied to compute the expected values and the variances of an inhomogeneous compound Poisson process. As in the homogeneous case, we examine the numerical accuracy of our approximating method, and compare it to the Monte Carlo simulation. The results from our method are in the 95% confidence interval of those from the Monte Calro simulation.

Finally, we discuss the application of our model to risk management in Section 3.7.2. An application of the recovery process model is to compute LGD in Basel Ⅱ for each company. LGD in Basel Ⅱ includes the expected cumulative LGD in a particular period (at least seven years). Because, in our model, we can calculate the expected cumulative recovery rate in an arbitrary period, our model is a useful tool for estimating the LGD. Moreover, we mention a method for calculating LGD in light of the variance and cost of the recovery.

According to paragraph 460 in Basel Committee on Banking Supervision (2004), LGD includes the entire cost for the recovery. Therefore, the recovery cost is important for credit risk management. However, because the data of the cost of the recovery is seldom available for academic study, its structure is unknown. Hence, we cannot model the recovery process with the recovery cost. Nevertheless, the recovery cost with which the lender completes the recovery for a short time may be less than that with which the lender does so for a long time. In that case, the recovery completion time that is derived in this dissertation is useful for the quantification of the recovery cost.

Two Companies Model In Chapter 4, we model the recovery process of the debts of two companies. We model the recovery processes for two defaulting companies using a bivariate homogeneous compound Poisson process and a bivariate inhomogeneous compound Poisson process. We assume that there are two types of shocks for the recoveries: the individual shocks that affect the recoveries of only one company and the common shocks that affect those of both companies. We also assume that there are delays between shock points and recovery points and that the delays are random variables. Then, the recovery points of two companies are correlated, but almost surely, the recoveries do not occur synchronously. As in Chapter 3, we also discuss two cases: models with and without interest rate.

In our inhomogeneous model, the number of recoveries of one company (marginal distribution of count process) is Poisson distributed with a nonconstant intensity. This corresponds to the modeling of the recovery process model for a single company. We note that the assumption of nonconstant intensity agrees with the empirical study by Itoh and Yamashita (2008) that the rate of change in recoveries is a deminishing function in time.

We derive the correlation of the recovery rates of the debts for two defaulting companies. Moreover, we derive the expected value and the standard deviation of the debt portfolio that comprises the debts of two companies.

For calculating these characteristics, we present two numerical methods of calculating the joint distribution function of the debts of two defaulting companies. One is based on the Vernic recursion formula and the other is based on the Monte Calro simulation. The Vernic recursion formula is a strong method to numerically calculate the distribution function of a bivariate compound Poisson distribution in the case where the debt does not bear interest. If the debt bears interest, there exists a correlation between the increments of the recoveries of two companies via interest and the Vernic recursion formula breaks down. Therefore, we use the Monte Carlo simulation in the model with interest. To the best of our knowledge, there is no study on the method of calculating the probability distribution of an inhomogeneous bivariate Poisson process by the Monte Carlo simulation, and we develop the procedure of the Monte Carlo simulation for calculating the same. We examine the numerical accuracy of the Monte Calro simulation in the non-interest case, and compare it to the Vernic recursion formula. The results from the Vernic recursion formula are in the 95% confidence interval of those from the Monte Calro simulation.

There is a constraint. Since it is impossible to identify the common and individual shocks from only the recovery data, we assume that the common and individual shocks are identified when shocks occur. If the data is complete, it means that the all the shock points and recovery points are present, and the estimation of the parameter of our model leads to identification. However, if the only recovery point is given, the estimation of the parameter of our model does not lead to identification.

In our model, the common shocks result in a correlation among the number of recoveries. Because of this structure of our model, only a nonnegative correlation of the two recovery rates is possible. This is the limitation of our model. This property is also pointed out by Bäuerle and Grübel (2005). In practice, the empirical correlation structure of the recovery process was not analyzed even by Itoh and Yamashita (2008). Therefore, the structure of the correlation of the empirical recovery rates in terms of the micro data is unknown. Nevertheless, we believe our model is useful to represent the basic features of the recovery rate of two companies in the real world.

Organization of this Dissertation The Panjer recursion formula and the Vernic recursion formula, which are methods for calculating the probability distribution function of the compound Poisson distribution and the bivariate compound Poisson distribution, are reviewed in Chapter 2. The recovery process model for a single defaulting company is mentioned in Chapter 3. The recovery process model for two defaulting companies is introduced in Chapter 4. Some technical issues are given in Chapter A.

Recovery Process Model for a Single Defaulting Company We introduce the single company model in Section 3.2 and the main results of Chapter 3 are given in Section 3.3. In Section 3.4, the distribution of the increment of recovery is specified and in Section 3.5, our model is extended to the inhomogeneous compound Poisson model and we suggest a new procedure for calculating the transition density of an inhomogeneous compound Poisson process. In Section 3.6, numerical results are demonstrated. We mention an application of our model to estimate LGD in Basel II in Section 3.7. The numerical example for the parameter estimation is shown in Section 3.8. In Section 3.9, the conclusion is given. In Section 2.1 and Section 2.2, we explain a homogeneous compound Poisson process and present the numerical methods of calculating the distribution function of compound Poisson distribution: the Panjer recursion formula and the fast Fourier transformation.

Recovery Process Model for Two Defaulting Companies Our two companies model is introduced in Section 4.2 and the main results of Chapter 4 are given in Section 4.3. The numerical procedures and the numerical results are demonstrated in Section 4.4. In Section 4.5, the discussion about the validity of fitting our model for the real recovery business, and the conclusion is given. More technical issues are provided in Section 2.3 and Section A.1.