Wavelet analysis for long memory processes

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Abstract

The discrete wavelet transform (DWT) has been used for long memory processes and their related topics. In this thesis, we explore the four topics on long memory processes via wavelet methods: the structure of wavelet coefficients of long memory processes, the new estimation procedure for the memory parameter, the estimation problem on long memory signal plus noise, and the analysis of multi-scale regression with long memory innovations and polynomial trends.

The analysis of long memory processes has been one of the major topics in time series analysis, financial econometrics, and other fields. The main character of these processes is that the autocovariance sequences decay slowly, so its spectral density function (SDF) has a pole at zero. In the time domain, because this property makes the covariance matrix dense, it is difficult to estimate the memory parameter by the maximum likelihood (ML) procedure. Therefore the estimator based on the conditional sum of squares (CSS) has been studied by many authors (Li and McLeod (1986), Robinson (1994), Sowell (1990), Hualde and Robinson (2011) and Nielsen (2011)). In the frequency domain, the log-periodogram regression (Geweke and Porter-Hudak (1983) and Robinson (1995b)) and the local Whittle estimation (Robinson (1995a) and Shimotsu and Phillips (2005)) has been studied.

The wavelet method introduced by Daubechies (1992) is one of the mathematical

tools to analyze signals, time series, images, multidimensional functional data and so on. In statistics, this method has been mainly used to analyze the topics which have the some characters in the time-frequency plane, like denoising, nonparametric regressions and long memory processes for these decades.

Especially, the wavelet analysis for long memory processes has been widely studied, because it can work well with not only the stationary case, i.e., $d < \frac{1}{2}$, but also the nonstationary case, $d \geq \frac{1}{2}$ if the degree of vanishing moments of wavelet filter is over $d - 1/2$; Tewfik and Kim (1992), Flandrin (1992) and McCoy and Walden (1996) show the approximation property of the wavelets for long memory processes; the approximated ML estimation (Jensen (2000) and Percival and Walden (2000)); the log-scale regression (Abry and Veitch (1998), Abry, Veitch, and Flandrin (1998) and Jensen (1999b)); the asymptotic properties of these estimators (Bardet (2002), Bardet, Lang, Moulines, and Soulier (2000), Moulines, Roueff, and Taqqu (2007), Moulines, Roueff, and Taqqu (2008) and Roueff and Taqqu (2009)), and applications for financial time series (Teyssière and Abry (2007)). These articles are based on the property that the wavelet coefficients of long memory processes become approximately white noise.

On the other hand, Jensen (1999a) and Craigmile, Guttorp, and Percival (2005) proposed the estimation procedures where the correlations of these coefficients are taken into consideration in order to improve the estimator. From the similar motivation, our first topic is to declare the model of the wavelet transformed ARFIMA processes, which is one of the long memory models, by using their generalized SDF. This is the similar way of Krim and Pesquet (1995) where the discrete wavelet transformed coefficients of the process with $d \in \mathbb{N}$ are considered, the wavelet filter is not specified and the degree of the MA term is counted indirectly. In contrast to Krim and Pesquet (1995), we assume that the wavelet filter is a $\mathrm{D}(L)$ filter, and then we can represent the SDF of the j-th wavelet coefficients directly and explore its model when $d \in \mathbb{R}$.

In the second topic, we discuss the "exact" wavelet-based ML estimation (EWE) for long memory processes which is inspired by Shimotsu and Phillips (2005). To take into account the dependence among the wavelet coefficients of long memory processes, Jensen (2000) used the 2D-wavelet transformation of the covariance matrix of the stationary ARFIMA model, and Craigmile, Percival, and Guttorp (2001) and Craigmile, Guttorp, and Percival (2005) tried to capture that dependence by using an AR model. In contrast to these studies, the EWE is calculated like the CSS estimator, and avoids two problems: one is the approximation problem of wavelet coefficients, and the other is the non-boundary coefficients problem. We also explore the consistency of the EWE.

In the third topic, we propose the estimation procedure for the memory parameter d of the long memory process plus noise model. The estimation of the memory parameter of the unobserved long memory process from the observed noisy process is one of the measurement error problems and, in the wavelet-domain, Wornell and Oppenheim (1992), Zhang, Bao, and Wu (2004) and Tanaka (2004) tried this problem. Because the existence of the noise term prevents the exact method proposed from estimating the memory parameter, we can not apply the exact method described in the second topic to the problem. Therefore, instead of the exact method, using the AR(1) approximation of the wavelet transformed long memory process at each level, which is introduced by Craigmile, Guttorp, and Percival (2005), we apply the ARMA(1,1) model to the wavelet coefficients of the observed process at each level. We compare the performance of the estimators of this procedure with those of the usual white noise approximation procedure as in Tanaka (2004) in numerical simulations.

In the last topic, we explore the multi-scale regression model with long memory innovations and trend terms, which is one of the extensions of the time-scale regression described in Appendix. The regression with long memory errors has been studied by several authors. In the time domain, Yajima (1988, 1991) introduced this model for the case that the regressors are nonstochastic (ex. polynomial trend) and the innovation is a stationary long memory process, i.e., $\{\varepsilon_t\} \sim \mathrm{I}(d_\varepsilon)$ with $0 < d_\varepsilon < \frac{1}{2}$. In the frequency domain method, Robinson and Hidalgo (1997) declared the central limit theorems of the estimator for the slope coefficient vector in the multiple regression model. Recently, in the time domain, Tsay (2007) considered the case that at least one of the regressors and innovations is nonstationary long memory process by using the multiple-differenced estimator which is calculated by the multiple-differenced transformation of the original regression model. In the wavelet domain, Fan and Whitcher (2003) consider the test statistics for spurious regression and Wu (2006) reported the Monte Carlo simulation study and applied feasible GLS when the regressor and the innovation process are stationary long memory processes. In this topic, we explore the asymptotic properties of the slope estimator in the wavelet domain.