Abstract

This dissertation focuses on the consumption-investment problem of an individual investor and aims to analyze and to answer the subsequent significant economic and financial questions. Starting from the very basic concern of macroeconomic problem, it studies what can explain consumption and consumption growth. Moving on to investment-concerned problems, one of the most important questions of the study is, perhaps, to answer how an investor should allocate financial assets to single securities within each asset class and among different asset classes. Besides, this research shows how the consumption growth and optimal investment strategies depend on certain internal and external influencers. The study also investigates the welfare loss occurring when an investor loses if he or she does not follow the optimal strategy and does not include some asset classes into consideration. Finally, the study aims at examining whether famous recommendations of investment advisors consistent with optimal investments derived in this dissertation.

In Chapter 2, important frameworks on basic financial instruments, stocks and bonds, and their price processes are reviewed. The Chapter also provides backgrounds on the dynamic programming approach and the use of the Hamilton-Jacobi-Bellman (HJB) equation as foundations to examine consumption growth and to solve optimal consumption-investment problems in later chapters.

Chapter 3 derives a Keynes-Ramsey rule (KRR) and an expected growth rate of consumption for a simple optimal consumption-investment problem with constant investment opportunities by using the HJB equation. The deterministic part of the KRR

is typically the average consumption growth rate, containing two constants and one time-varying term. For more clarification, it is a combination of the usual difference between the interest rate and time preference rate, the term capturing the impact of market prices of risk, and the time horizon related term which is the effect of introducing time horizon into the problem. The first two terms are also the KRR for the infinite time horizon model while the addition of the last term creates the KRR for the finite model. That the average growth also depends on this function confuses an analysis of the interest rate effect on consumption growth. However, the study finds that interest rate movements not only positively affect consumption growth, but also indirectly and negatively affect the growth via the market price of risks. It also finds an inverse relationship between time horizon and both the level and the growth rate of consumption. Moreover, the strategy recommends a negative relationship between risk-aversion factor and consumption growth, implying that a more risk-averse individual is more-contented with current consumption. In search of the KRR for a more complex dynamic asset allocation problem with stochastic interest rates, we see that the main challenge is to find an optimal consumption-investment strategy which plays an important role in economic interpretation in this case.

Chapter 4 investigates an explicit investment policy with hedged variations of mixed bond-stock-commodity dynamic portfolio problems under a simple interest rate model and mean-reverting commodity prices. The findings suggest that the optimal allocation to a zero-coupon bond and a commodity is a combination of speculative terms and hedge terms as protection of change in interest rates and change in commodity market price of risk, respectively. The allocation to stocks, however, depends only on the

speculative portfolio as there is no need for risk protection. The policy recommends a negative relationship between risk-aversion factor and riskier assets, stocks and commodities, while it proposes a positive relationship to that of zero-coupon bonds. This is consistent with the professional advice that investors who can tolerate more risk should invest more in riskier assets such as stocks and commodities. The Chapter also finds inverse relationships between commodity prices and positions in the stock and the commodity but no conclusions can be made regarding the direction of zero-coupon bond investment from a rise in commodity prices. The welfare loss due to negligence of commodity investment is also solved in explicit form.

Chapter 5 provides numerical examples to support ideas developed in previous chapters. For the topic related to consumption growth, two important conclusions can be reached. The existence of the time-varying term is great when consumption-investment time horizon is short and investors are more afraid of risks. Moreover, interest rates have a negative relationship with both infinite and finite consumption growth and the effect is especially substantial when investors are less risk-averse. The second result is against previous theoretical and empirical studies.

Regarding to a numerical study of mixed bond-stock-commodity, the study finds that bond investment has a positive relationship with the time horizon. Considering commodity investment, investors, instead, tend to decrease their portions as investment horizon is longer but will eventually increase allocation in the long run. Besides, conventional investors slowly increase their portions in commodity allocation compared to aggressive investors. Secondly, if the spot commodity price has a significant negative effect on the risk premium, positions in risky assets decrease with increasing commodity

prices. Moreover, conservative investors smoothly reduce their positions compared to aggressive investors. Also, reduction in risky assets implies increasing position in the bank account. Finally, the study of correlation and financial allocation results in various outcomes depending on estimated parameters. In summary, allocation in each asset will be increased when relevant correlations convert to -1.